2. Which of the following is a negation for "All dogs are loyal"? More than one answer may be correct.
   a. All dogs are disloyal.
   b. No dogs are loyal.
   c. Some dogs are disloyal.
   d. Some dogs are loyal.
   e. There is a disloyal animal that is not a dog.
   f. There is a dog that is disloyal.
   g. No animals that are not dogs are loyal.
   h. Some animals that are not dogs are loyal.

3. Write a formal negation for each of the following statements:
   a. $\forall$ fish x, x has gills.
   b. $\forall$ computers c, c has a CPU.
   c. $\exists$ a movie m such that m is over 6 hours long.
   d. $\exists$ a band b such that b has won at least 10 Grammy awards.

4. Write an informal negation for each of the following statements. Be careful to avoid negations that are ambiguous.
   a. All dogs are friendly.
   b. All people are happy.
   c. Some suspicions were substantiated.
   d. Some estimates are accurate.

5. Write a negation for each of the following statements.
   a. Any valid argument has a true conclusion.
   b. Every real number is positive, negative, or zero.

6. Write a negation for each of the following statements.
   a. Sets A and B do not have any points in common.
   b. Towns P and Q are not connected by any road on the map.

7. Informal language is actually more complex than formal language. For instance, the sentence "There are no orders from store A for item B" contains the words *there are*. Is the statement existential? Write an informal negation for the statement, and then write the statement formally using quantifiers and variables.

8. Consider the statement "There are no simple solutions to life's problems." Write an informal negation for the statement, and then write the statement formally using quantifiers and variables.

Write a negation for each statement in 9 and 10.

9. $\forall$ real numbers x, if $x > 3$ then $x^2 > 9$.

10. $\forall$ computer programs P, if P compiles without error messages, then P is correct.

In each of 11–14 determine whether the proposed negation is correct. If it is not, write a correct negation.

11. *Statement:* The sum of any two irrational numbers is irrational.
    Proposed negation: The sum of any two irrational numbers is rational.

12. *Statement:* The product of any irrational number and any rational number is irrational.
    Proposed negation: The product of any irrational number and any rational number is rational.

13. *Statement:* For all integers n, if n^2 is even then n is even.
    Proposed negation: For all integers n, if n^2 is even then n is not even.

14. *Statement:* For all real numbers x_1 and x_2, if $x_1^2 = x_2^2$ then $x_1 = x_2$.
    Proposed negation: For all real numbers x_1 and x_2, if $x_1^2 = x_2^2$ then $x_1 \neq x_2$.

15. Let $D = \{-48, -14, -8, 0, 1, 3, 16, 23, 26, 32, 36\}$. Determine which of the following statements are true and which are false. Provide counterexamples for those statements that are false.
    a. $\forall x \in D$, if x is odd then $x > 0$.
    b. $\forall x \in D$, if x is less than 0 then x is even.
    c. $\forall x \in D$, if x is even then $x \leq 0$.
    d. $\forall x \in D$, if the ones digit of x is 2, then the tens digit is 3 or 4.
    e. $\forall x \in D$, if the ones digit of x is 6, then the tens digit is 1 or 2.

In 16–23, write a negation for each statement.

16. $\forall$ real numbers x, if $x^2 \geq 1$ then $x > 0$.

17. $\forall$ integers d, if $6/d$ is an integer then $d = 3$.

18. $\forall x \in \mathbf{R}$, if $x(x+1) > 0$ then $x > 0$ or $x < -1$.

19. $\forall n \in \mathbf{Z}$, if n is prime then n is odd or $n = 2$.

20. $\forall$ integers a, b and c, if $a - b$ is even and $b - c$ is even, then $a - c$ is even.

21. $\forall$ integers n, if n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

22. If the square of an integer is odd, then the integer is odd.

23. If a function is differentiable then it is continuous.

24. Rewrite the statements in each pair in if-then form and indicate the logical relationship between them.
    a. All the children in Tom's family are female.
       All the females in Tom's family are children.
    b. All the integers that are greater than 5 and end in 1, 3, 7, or 9 are prime.
       All the integers that are greater than 5 and are prime end in 1, 3, 7, or 9.

25. Each of the following statements is true. In each case write the converse of the statement, and give a counterexample showing that the converse is false.
    a. If n is any prime number that is greater than 2, then $n + 1$ is even.
    b. If m is any odd integer, then $2m$ is even.
    c. If two circles intersect in exactly two points, then they do not have a common center.