Practice Proof Problems

Prove the following theorems using direct proof.

1. If x and y are both even integers then the product xy is an even integer.
2. If n is an even integer, 4 <= n <=12 then n is the sum of two prime numbers.
3. The sum of two odd integers is even.
4. If an integer p is even then so is p+2.
5. The square of an even number is divisible by 4.

Prove the following theorems using proof by contrapositive.

1. For all integers n if n^2 is even then n is even
2. If the product of two positive real numbers is greater than 400 then at least one of the numbers is greater than 10.
3. For all integers m and n, if m+n is even then m and n are either both odd or both even.
4. If a number x is positive then so is x+1.
5. For integers a, b, and c, if ab is not divisible by c then a is not divisible by c and b is not divisible by c. (NOTE: If x is divisible by y this means that x/y produces some integer z.)

Prove the following theorems using proof by contradiction.

1. If a number added to itself gives itself then the number is zero.
2. The product of odd integers is never even.
3. The square root of any irrational number is irrational.
4. The product of any nonzero rational number and any irrational number is irrational
5. For all integers m and n, if m+n is even then m and n are either both odd or both even.

Prove the following theorems using proof by cases.

1. The expression $2m^2 – 1$ is odd for all integers m. [2 cases]
2. If x is a real number then | x+3 | - x > 2 [2 cases]
3. If x is a real number then |x-1| + | x + 5 | >= 6 [3 cases]
4. If n is an even integer then n = 4j or n = 4j – 2 [2 cases. Hint, if you say n=2m then the two cases deal with the two possibilities for m]
5. CHALLENGE - If a and b are real numbers than | |a| - |b| | <= | a-b | [6 cases]