Terminology:

problem - question we seek an answer for, e.g., "what is the largest item in a list/array?"

parameters - variables with unspecified values

problem instance - assignment of values to parameters, i.e., the specific input to the problem

	0	1	2	3	4	5	6
myList:	5	10	2	15	20	1	11

largest: ?

n:	7

(number of elements)

algorithm - step-by-step procedure for producing a solution

basic operation - fundamental operation in the algorithm (i.e., operation done the most) Generally, we want to derive a function for the number of times that the basic operation is performed related to the *problem size.*

problem size - input size. For algorithms involving lists/arrays, the problem size is the number of elements ("n").

```
import time

def main():
    aList = range(1,1000001)
    start = time.time()
    sum = sumList(aList)
    end = time.time()
    print "Time to sum the list was %.3f seconds" % (end-start)

def sumList(myList):
    """Returns the sum of all items in myList"""
    total = 0
    for item in myList:
        total = total + item
    return total

main()
```

a) What is the basic operation? addition in line "`total = total + item`"

b) What is the problem size? # of items in `myList`

c) What would determine how fast this algorithm would run?

- Problem size - # items in the list
- speed of the computer

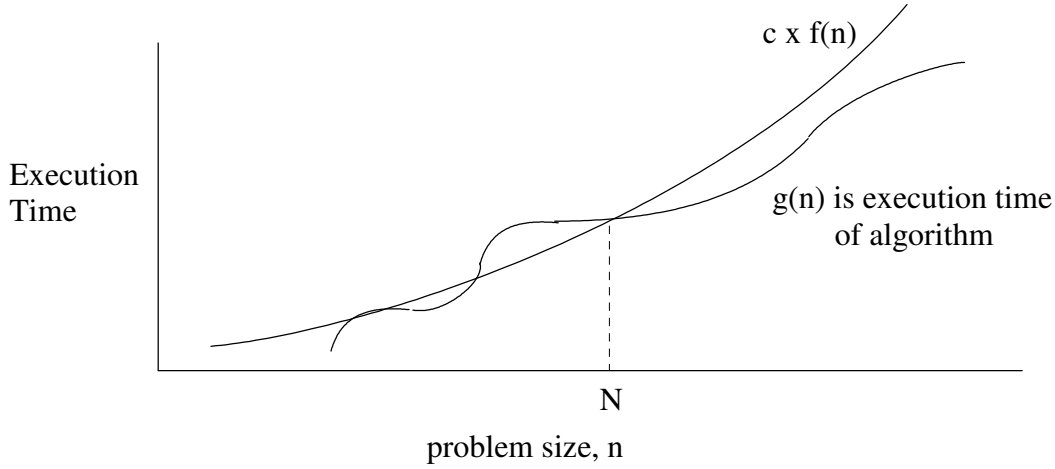
Execution time, T(n), of sumList = (time to perform code which is done once) + n x (time to perform loop once).
Computers of different speeds would not effect this formula, but only the constants:

- time to perform code which is done once, and
- time to perform loop once

The big-oh and big-theta definitions that follow provide a computer independent measure of an algorithm's performance.

Big-oh Definition - asymptotic upper bound
For a given complexity function $f(n)$, $O(f(n))$ is the set of complexity functions $g(n)$ for which there exists some positive real constant c and some nonnegative integer N such that for all $n \geq N$,

$$g(n) \leq c \times f(n).$$



$T(n) = c_1 + c_2\, n = 100 + 10\, n$ is $O(n)$.

"Proof": Pick c = 110 and N = 1, then $100 + 10\, n \leq 110\, n$ for all $n \geq 1$.

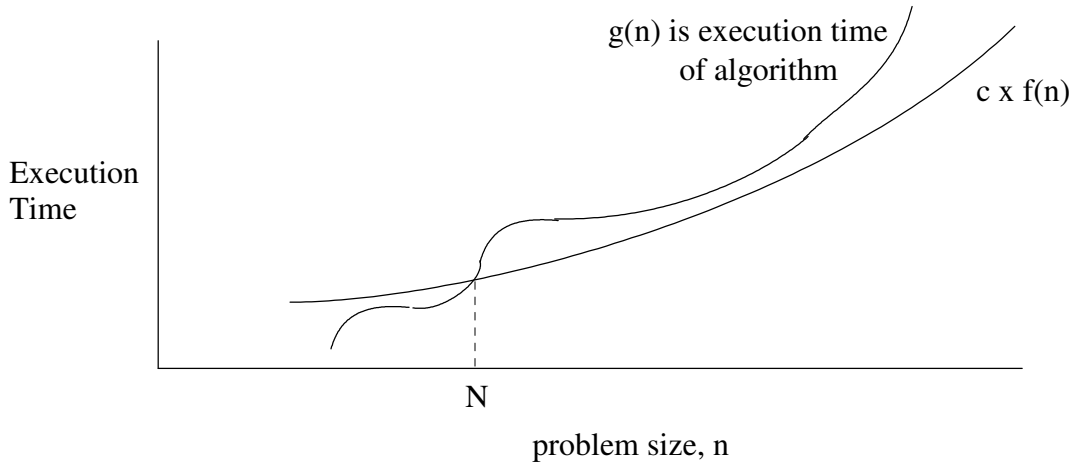
$$100 + 10\, n \leq 110\, n$$
$$100 \leq 100\, n$$
$$1 \leq n$$

Problem with big-oh:

If T(n) is $O(n)$, then it is also $O(n^2)$, $O(n^3)$, $O(n^3)$, $O(2^n)$, ... since these are also upper bounds.

Omega Definition - asymptotic lower bound
For a given complexity function $f(n)$, $\Omega(f(n))$ is the set of complexity functions $g(n)$ for which there exists some positive real constant c and some nonnegative integer N such that for all $n \geq N$,

$$g(n) \geq c \times f(n).$$



2. Let $T(n) = c_1 + c_2 n = 100 + 10 n$. Show that $100 + 10 n$ is $\Omega(n)$.

"Proof": We need to find a c and N so that the definition is satisfied, i.e., $100 + 10 n \geq c n$ for all $n \geq N$.

What c and N will work?

I would pick c = 10, so we need to find an N such that $100 + 10 n \geq 10 n$ for all $n \geq N$. By picking c = 10, this will be easy since

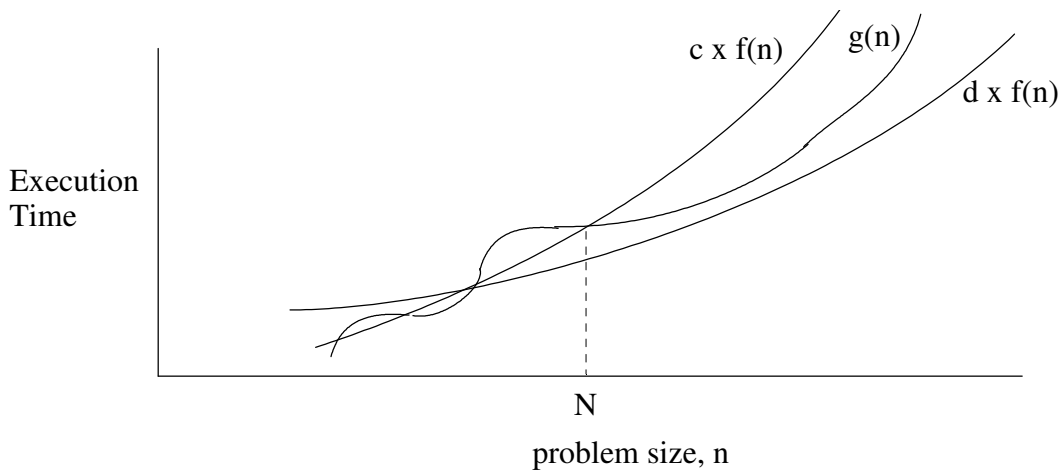
$$100 + 10 n \geq 10 n$$
$$100 + 10 n - 10 n \geq 10 n - 10 n$$
$$100 \geq 0 \text{ which is always true for an value of n}$$

I'll pick N = 1.

(NOTE: Other values of c would work too, like c = 9, c = 8, ..., c = 1)

Theta Definition - asymptotic upper and lower bound, i.e., a "tight" bound or "best" big-oh
For a given complexity function $f(n)$, $\theta(f(n))$ is the set of complexity functions $g(n)$ for which there exists some positive real constants c and d and some nonnegative integer N such that for all $n \geq N$,

$$d \times f(n) \leq g(n) \leq c \times f(n).$$



$T(n) = c_1 + c_2 n = 100 + 10 n$ is $\theta(n)$ since it is both $O(n)$ and $\Omega(n)$.

3. Suppose that you have an $\theta(n^2)$ algorithm that required 10 seconds to run on a problem size of 1000. How long would you expect the algorithm to run on a problem size of 10,000?

A $\theta(n^2)$ algorithm implies execution time, $T(n) = c_1 n^2 +$ (slower growing terms like $c_2 n + c_3$), where c_1, c_2, c_3 are constants determined by the speed of your computer.

For large n's, $T(n) \approx c_1 n^2$. We are given that the problem requires 10 seconds to run on a problem size of 1000, so $T(1000) = 10$ seconds $\approx c_1 1000^2$, solving for $c_1 = 10$ seconds / $1000^2 = 10$ seconds / $10^6 = 1$ second / $10^5 = 10^{-5}$ seconds. For a larger size problem, c_1 remains constant on the same computer.

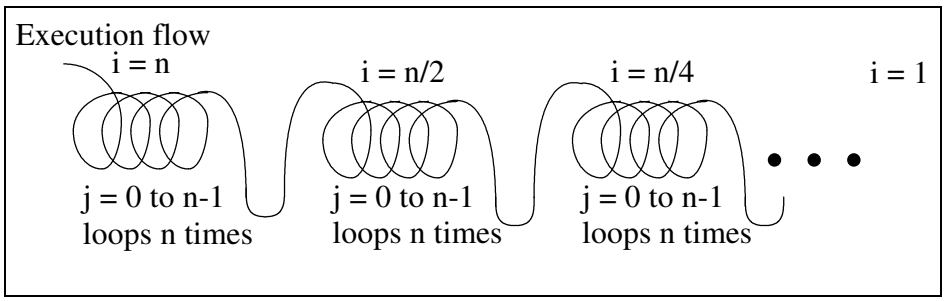
Thus, $T(10{,}000) \approx c_1 10000^2 = 10^{-5}$ seconds x $10000^2 = 1000$ seconds.

The problem size got 10 times bigger -- it went from 1000 to 10,000. However, the execution time got 100 times longer -- it went from 10 seconds to 1000 seconds, since the algorithm was $\theta(n^2)$.

4. Analyze the below algorithm to determine its theta notation, $\theta(\ )$.

```
i = n
while i > 0:
     for j in xrange(n):
         # something of O(1)

     # end for
     i = i / 2
# end while
```



The outer while-loop executes $\theta(\log_2 n)$ times. IMPORTANT: Any time n is repeatedly halved down to 1 or 0, the loop executes $\theta(\log_2 n)$ times. Similiarly, any time a variable i starts at 1 and is repeatedly doubled to reach the value n, the loop executes $\theta(\log_2 n)$ times.

For each iteration of the outer-loop, the inner-loop executes n times, so the whole algorithm is $\theta(n \log_2 n)$. Pronounced as "theta of n log n".

5.

```
def linearSearch(target, aList):
    """Returns the index position of target in aList or -1 if target
    is not in aList"""
    for position in xrange(len(aList)):
        if target == aList[position]:
            return position
    return -1
```

a) For sequential search, what is the best-case time complexity B (n)? In the best-case the target value is in position 0 of the list, so the for-loop stops after one iteration. This will take a constant amount of time so the best-case time complexity is $\theta(1)$, which is "constant time".

b) For sequential search, what is the worst-case time complexity W (n)? In the worst-case the target is NOT in the list so the for-loop must loop through all n list positions. Thus, the worst-case time complexity θ(n), which is "linear".

c) If the probability of a successful sequential search is p, then what is the probability on an unsuccessful search? 1-p

d) If the probability of a successful sequential search is p, then what is the probability of finding the target value at a specific index in the array? All thinks being equal, we can spread the probability p across all n positions in the list, so the probability of a target a specific index would be p/n.

e) For each position in the list of size n, how many comparisons would be needed for a successful search?

	0	1	2	3 . . .	n-1	All unsuccessful searches
# compares:	1	2	3		n	n
probability:	$\frac{p}{n}$	$\frac{p}{n}$	$\frac{p}{n}$		$\frac{p}{n}$	(1-p)

e) Write a summation for the average number of comparisons.

$$\frac{p}{n}1 + \frac{p}{n}2 + \frac{p}{n}3 + ... + \frac{p}{n}n + (1-p)n$$

$$= \frac{p}{n}(1+2+3+...+n) + (1-p)n$$

$$= \frac{p}{n}\frac{n}{2}(n+1) + (1-p)n$$

$$= \frac{p}{2}(n+1) + (1-p)n$$

$$= \frac{p}{2}n + \frac{p}{2} + n - pn$$

$$= n - \frac{p}{2}n + \frac{p}{2}$$

$$= n(1-\frac{p}{2}) + \frac{p}{2}$$

f) What is the average time complexity, A (n)?

$n(1-\frac{p}{2}) + \frac{p}{2}$ which is θ(n) since any given p causes $(1-\frac{p}{2})$ to be a constant.