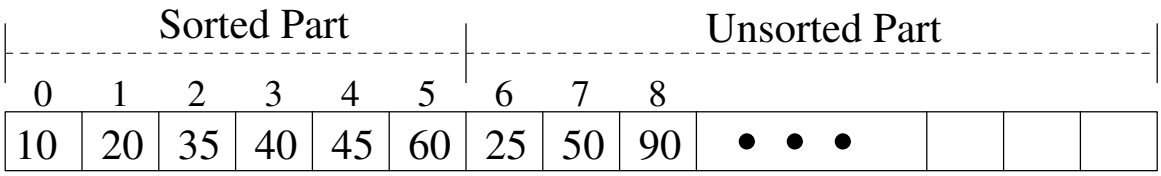
Homework #2 Data Structures
Due: February 4, 2010 (Thursday at 5 PM)

Homework #2 is a pencil-and-paper assignment involving algorithm/program analysis. Answer the questions for each of the following algorithms.

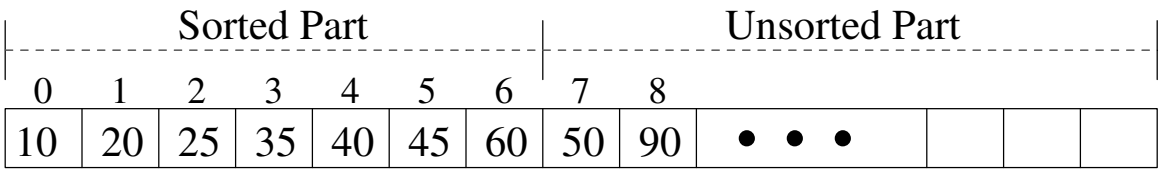
1. Another simple sort is called insertion sort. Recall that in a simple sort:

- the outer loop keeps track of the dividing line between the sorted and unsorted part with the sorted part growing by one in size each iteration of the outer loop.
  - the inner loop's job is to do the work to extend the sorted part's size by one.

After several iterations of insertion sort's outer loop, an array might look like:



In insertion sort the inner-loop takes the "first unsorted element" (25 at index 6 in the above example) and "inserts" it into the sorted part of the list "at the correct spot." After 25 is inserted into the sorted part, the array would look like:



Code for insertion is given below:

```
void insertionSort(int array[], int size) {

      int firstUnsortedIndex, testIndex, elementToInsert;

      for  (firstUnsortedIndex = 1; firstUnsortedIndex < size; firstUnsortedIndex++) {

            elementToInsert = array[firstUnsortedIndex];
            testIndex = firstUnsortedIndex-1;

            while (testIndex >=0 && array[testIndex] > elementToInsert ) {
                  array[ testIndex + 1 ] = array[ testIndex ];
                  testIndex =  testIndex - 1;
            } // end while

            array[ testIndex + 1 ] = elementToInsert;

      } // end for

} // end insertionSort
```

To help you analyze the above algorithm to determine its **worst-case** theta notation, $\Theta(\ )$, complete the following table like we did in class to determine how many times the inner-loop executes.

a)

`firstUnsortedIndex:`	**1**	**2**	**3**	**. . .**	**size-2**	**size-1**
Range of `testIndex` values				**. . .**		
Number of times inner-loop executes				**. . .**		

b) Write a summation formula for the total number of times that the inner-loop executes.

c) What is the overall worst-case $\Theta(\ )$ notation for insertion sort?

d) What is the worst-case $\Theta(\ )$ notation for the number of element moves?

e) What is the worst-case $\Theta(\ )$ notation for the number of element comparisons?

f) What is the best-case $\Theta(\ )$ notation for the number of element moves?

g) What is the best-case $\Theta(\ )$ notation for the number of element comparisons?

2. Consider the following alternative coding of insertion sort which utilizes an insert function.

```
void insert(int numbers[], int elementToInsert, int firstSortedIndex) {
      int testIndex;

      testIndex = firstSortedIndex;

      while (testIndex >=0 && numbers[testIndex] > elementToInsert ) {
            numbers[ testIndex + 1 ] = numbers[ testIndex ];
            testIndex =  testIndex - 1;
      } // end while

      numbers[ testIndex + 1 ] = elementToInsert;
} // end Insert

void insertionSort(int numbers[], int length) {
      int firstUnsortedIndex;

      for  (firstUnsortedIndex = 1; firstUnsortedIndex < length; firstUnsortedIndex++) {

            insert(numbers, numbers[firstUnsortedIndex], firstUnsortedIndex-1);

      } // end for
} // end insertionSort
```

a) Since the insertionSort function only calls Insert, does this improve the worst-case $\Theta(\ )$ notation. Explain your answer.

b) What implications does your answer in part (a) have for analyzing large programs that are split into many functions?

3. Consider a third alternative of insertion sort (no code provided) which utilizes the fact that the "sorted" part of the array is sorted. The previous two versions of insertion sort linearly/sequentially scanned the sorted part of the array for the insertion point. In this problem consider the performance improvements achieved by performing a binary search on the sorted part of the array to find the spot to insert the "first unsorted elemen

Answer the following questions about this **third alternative of insertion sort** (you do NOT need to write the code for this algorithm):

a) What is the worst-case $\Theta(\ )$ notation for the number of element moves? (justify your answer)

b) What is the worst-case $\Theta(\ )$ notation for the number of element comparisons? (justify your answer)

c) What is the overall worst-case $\Theta(\ )$ notation for this third insertion sort? (justify your answer)

d) What is the best-case $\Theta(\ )$ notation for the number of element moves? (justify your answer)

e) What is the best-case $\Theta(\ )$ notation for the number of element comparisons? (justify your answer)

f) What is the overall best-case $\Theta(\ )$ notation for this third insertion sort? (justify your answer)