1. A priority queue has the same operations as a regular queue, except the items are NOT returned in the FIFO (first-in, first-out) order. Instead, each item has a priority that determines the order they are removed. A hospital emergency room operates like a priority queue -- the person with the most serious injury has highest priority even if they just arrived.

a) Suppose that we have a priority queue with integer priorities such that the smallest integer corresponds to the highest priority. For the following priority queue, which item would be dequeued next?

```
priority queue: 40 10 79
               30 13
               5
```

b) To implement a priority queue, we could use an unordered Python list. If we did, what would be the worst-case theta (Θ()) notation for each of the following methods: (justify your answer)

- enqueue:
- dequeue:

c) To implement a priority queue, we could use a linked list ordered by priorities, e.g., the LinkedPriorityQueue class of chapter 15. If we did, what would be the worst-case theta (Θ()) notation for each of the following methods: (justify your answer)

- enqueue:
- dequeue

2. Sections 18.9 - 18.11 discuss a very “non-intuitive”, but powerful list/array-based approach to implement a priority queue, call a heap. The list/array is used to store a complete binary tree (a full tree with any additional leaves as far left as possible) with the items being arranged by heap-order property, i.e., each node is less than either of its children. An example of a min heap “viewed” as a complete binary tree would be:

```
[0]
  6
 /   \
[1]   [2]
 15   10
 / \
114 20 20 50
 / \ / \ / \
[7] [8] [9] [10]
 300 125 117
```


a) For the above heap, the list/array indexes are indicated in [ ]'s. For a node at index $i$, what is the index of:
- its left child if it exists:
- its right child if it exists:
- its parent if it exists:

b) What would the above heap look like after adding 13 and then 8?

c) What is the worst-case theta ($\Theta()$) notation for adding a new item in the heap?

d) What is the worst-case theta ($\Theta()$) notation for removing the smallest item from the heap?