Name:_____

Consider the coin-change problem: Given a set of coin types and an amount of change to be returned, determine the **fewest number** of coins for this amount of change.

1) For coins of {1, 5, 10, 12, 25, 50}, typical timings:

Change Amount	Run-Time (seconds)	Number of Tree Nodes
200	0.92	236,583
300	33.23	8,617,265
320	64.12	16,676,454
340	116.8	30,370,729

a) Why the exponential growth in run-time?

2) As with Fibonacci, the coin-change problem can benefit from dynamic program since it was slow due to solving the same problems over-and-over again. Recall the general idea of dynamic programming:

- Solve smaller problems before larger ones
- store their answers
- look-up answers to smaller problems when solving larger subproblems

Advantage:
Eliminates redundant work since each smaller problem solved only once!

a) How do we solve the coin-change problem using dynamic programming?

Name:_____

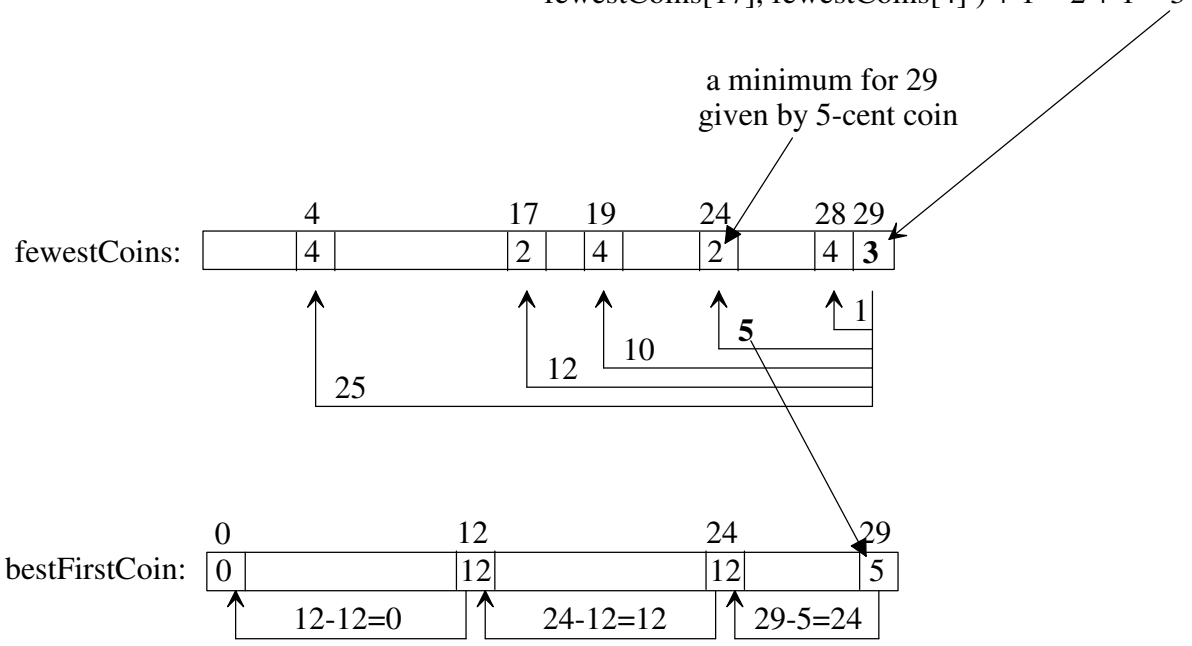
Dynamic Programming Coin-change Algorithm:

I. Fills an array fewestCoins from 0 to the amount of change. An element of fewestCoins stores the fewest number of coins necessary for the amount of change corresponding to its index value.

For 29-cents using the set of coin types {1, 5, 10, 12, 25, 50}, the dynamic programming algorithm would have previously calculated the fewestCoins for the change amounts of 0, 1, 2, ..., up to 28 cents.

II. If we record the best, first coin to return for each change amount (found in the "minimum" calculation) in an array bestFirstCoin, then we can easily recover the actual coin types to return.

fewestCoins[29] = minimum(fewestCoins[28], fewestCoins[24], fewestCoins[19],
fewestCoins[17], fewestCoins[4]) + 1 = 2 + 1 = 3



Extract the coins in the solution for 29-cents from bestFirstCoin[29], bestFirstCoin[24], and bestFirstCoin[12]

b) Extend the lists through 32-cents.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
fewestCoins:	0	1	2	3	4	1	2	3	4	5	1	2	1	2	3	2	3	2	3	4	2	3	2	3	2	1	2	3	4	3			

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
bestFirstCoin:	0	1	1	1	1	5	1	1	1	1	10	1	12	1	1	5	1	5	1	1	10	1	10	1	12	25	1	1	1	5			

c) What coins are in the solution for 32-cents?