1) Consider the following DNA sequence with restriction sites marked. The complete digest is as shown.

\[ \text{DNA} \]

\[ \begin{array}{cccccc}
0 & 2 & 4 & 7 & 10 \\
\hline
2 & 2 & 3 & 3 & \\
\end{array} \]

a) The *partial digest* would generate fragments of lengths between all restriction sites. Complete the following diagram showing the partial digest.

\[ \text{DNA} \]

\[ \begin{array}{cccccc}
0 & 2 & 4 & 7 & 10 \\
\hline
2 & 2 & 3 & 3 & \\
\end{array} \]

b) A *multiset* is a set that allows duplicates. What would be the multiset for the lengths of DNA fragments for the above partial digest?

2) If \( X = \{ x_1, x_2, x_3, \ldots, x_n \} \) is a set of \( n \) points on a line segment in increasing order, the \( \Delta X \) denotes the multiset of all pairwise distances between points in \( X \).

a) For \( X = \{ 0, 2, 4, 7, 10 \} \) as in the above DNA, what is \( \Delta X \)?

b) If the set \( X \) is stored in an array \( X[1..n] \), write a nested loops to generate \( \Delta X \).

c) Develop a formula between \(|X|\) and \(|\Delta X|\).
3) Consider the following brute-force algorithm for solving the partial digest problem (PDP).

1. \textbf{BruteForcePDP}(L, n):
2. \hspace{1cm} M \leftarrow \text{maximum element in } L
3. \hspace{1cm} \text{for every set of } n - 2 \text{ integers } 0 < x_2 < \ldots < x_{n-1} < M
4. \hspace{1cm} X \leftarrow \{0, x_2, \ldots, x_{n-1}, M\}
5. \hspace{1cm} \text{Form } D_X \text{ from } X
6. \hspace{1cm} \text{if } D_X = L
7. \hspace{1cm} \hspace{1cm} \text{return } X
8. \hspace{1cm} \text{output “no solution”}

a) In line 4, why does the set \( X \) contain 0 and \( M \)?

b) To analyze the algorithm to determine its \( O(\cdot) \) notation, draw a “search tree” of all sets of \( n-2 \) integer.

4) How could we be smarter about generating the possible sets of \( X \)? Hint: Use more information about the specific problem instance.