

Discrete Structures Test 1

Question 1. (8 points) Complete the following truth tables.

A	B	$A \vee B$	$A \wedge B$	$A \rightarrow B$	$A \leftrightarrow B$
T	T	T	T	T	T
T	F	T	F	F	F
F	T	T	F	T	F
F	F	F	F	T	T

Question 2. (10 points) Use a truth table to show that the following wff is a tautology.

$$[(P \rightarrow Q) \wedge Q'] \rightarrow P'$$

P	Q	$P \rightarrow Q$	Q'	$(P \rightarrow Q) \wedge Q'$	P'	$[(P \rightarrow Q) \wedge Q'] \rightarrow P'$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Question 3. (12 points) Let A, B, C, and D be the following statements:

- A: The villain is French.
- B: The hero is American.
- C: The heroine is British.
- D: The movie is good.

a) Translate the following compound statements into propositional well-formed formulas (wffs).

i. The hero is American and the movie is good. $B \wedge D$

ii. If the movie is good, then either the hero is American or the heroine is British.

$$D \rightarrow (B \vee C)$$

iii. The hero is not American, but the villain is French.

$$B' \wedge A$$

b) Use A, B, C, and D as defined above to translate the following statements into English.

i. $B \vee C'$ The hero is American or the heroine is not British.

ii. $B' \vee (A \rightarrow C)$ Either the hero is not American, or if the villain is French then the heroine is British.

iii. $(C \wedge A') \leftrightarrow B$ The heroine is British and the villain is not French if and only if the hero is American.

Question 4. (20 points) Use propositional logic, prove that the following argument is valid.

$$A \wedge (B \rightarrow C) \wedge [(A \wedge B) \rightarrow (D \wedge C')] \rightarrow [B \rightarrow D]$$

1	A	hyp
2	$B \rightarrow C$	hyp
3	$(A \wedge B) \rightarrow (D \wedge C')$	hyp
4	B	hyp, by the deduction method
5	$A \wedge B$	1, 2, con
6	$D \wedge C'$	5, 3, mp
7	D	6, sim

Question 5. (15 points) Using the statement letters E, Q, and B, translate the following English argument into propositional logic. (YOU DO **NOT** NEED TO PROVE THE ARGUMENT IS VALID!)

If the program is efficient, it executes quickly. Either the program is efficient, or it has a bug. However, the program does not execute quickly. Therefore, it has a bug.

$$(E \rightarrow Q) \wedge (E \vee B) \wedge Q' \rightarrow B$$

Question 6. (10 points) Give an interpretation to prove that the following wff is not valid.

$$[(\forall x)P(x) \vee (\exists x)Q(x)] \rightarrow (\forall x)[P(x) \vee Q(x)]$$

The domain of interpretation for x is the integers

$P(x)$ means: $x > 6$

$Q(x)$ means: $x > 6$

Under this interpretation we have: $F \vee T \rightarrow F$ which is not valid.

Question 7. (15 points) Prove the following wff using predicate logic.

$$(\forall x)[M(x) \rightarrow P(x)] \wedge (\exists x)[S(x) \wedge M(x)] \rightarrow (\exists x)[S(x) \wedge P(x)]$$

1	$(\forall x)[M(x) \rightarrow P(x)]$	hyp
2	$(\exists x)[S(x) \wedge M(x)]$	hyp
3	$S(a) \wedge M(a)$	2, ei
4	$M(a) \rightarrow P(a)$	1, ui
5	$M(a)$	3, sim
6	$P(a)$	4, 5, mp
7	$S(a)$	3, sim
8	$S(a) \wedge P(a)$	6, 7, con
9	$(\exists x)[S(x) \wedge P(x)]$	8, eg

Question 8. (10 points) Using the predicate symbols

$H(x)$: x can hit the baseball a long way

$M(x)$: x can make a lot of money

$T(x)$: x is a member of the Titans

k is the constant for Ken

translate the following verbal argument into a predicate logic wff.

Everyone who can hit a baseball a long way can make a lot of money. Ken is a member of the Titans. Ken can hit a baseball a long way. Therefore, some member of the Titans can make a lot of money.

(YOU DO **NOT** NEED TO PROVE THE ARGUMENT IS VALID!)

$$(\forall x)[H(x) \rightarrow M(x)] \wedge T(k) \wedge H(k) \rightarrow (\exists x)[T(x) \wedge M(x)]$$