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## Homework \#3 Computer Organization

Due: Feb. 15, 2019 (Friday) by 3 PM

1. Recall that the sum-of-products (SOP) Boolean formula for the carry-out ( $c_{i+1}$ ) of a n-bit adder was :

1-bit adder: $\quad c_{i+1}=x_{i} y_{i}+x_{i} c_{i}+y_{i} c_{i}$.
2-bit adder: $\quad c_{i+1}=x_{i} y_{i}+x_{i}\left(x_{i-1} y_{i-1}+x_{i-1} c_{i-1}+y_{i-1} c_{i-1}\right)+y_{i}\left(x_{i-1} y_{i-1}+x_{i-1} c_{i-1}+y_{i-1} c_{i-1}\right)$
$=x_{i} y_{i}+x_{i} x_{i-1} y_{i-1}+x_{i} x_{i-1} c_{i-1}+x_{i} y_{i-1} c_{i-1}+y_{i} x_{i-1} y_{i-1}+y_{i} x_{i-1} c_{i-1}+y_{i} y_{i-1} c_{i-1}$
3-bit adder: $\quad c_{i+1}=x_{i} y_{i}+x_{i}(7$ product terms of 2-bit adder $)+y_{i}$ (7 product terms of 2-bit adder )
$=x_{i} y_{i}+x_{i} x_{i-1} y_{i-1}+x_{i} x_{i-1} x_{i-2} y_{i-2}+\ldots$ ( 11 product terms omitted $)+y_{i} y_{i-1} y_{i-2} c_{i-2}$
4-bit adder: $\quad c_{i+1}=x_{i} y_{i}+x_{i}(15$ product terms of 3-bit adder $)+y_{i}(15$ product terms of 3-bit adder )

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=x_{i} y_{i}+x_{i} x_{i-1} y_{i-1}+\ldots(28 \text { product terms omitted })+y_{i} y_{i-1} y_{i-2} y_{i-3} c_{i-3}
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a) Complete the following table showing the gate delays for different types of adders assuming a 9-input limit into any gate.

| Type <br> of <br> Adder | \# of product terms <br> in SOP expression <br> to be OR'ed | Most \# of inputs <br> in any of the <br> product terms | \# gate delays due <br> to product/AND <br> terms | \# gate delays <br> due to <br> sum/OR | Gate <br> Delay per <br> Adder | Gate Delays for <br> 32-bit ripple adder <br> using adders of this <br> type |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1-bit | 3 | 2 | 1 | 1 | 2 | $32 \times 2=64$ |
| 2-bit | 7 | 3 | 1 | 1 | 2 | $16 \times 2=32$ |
| 3-bit | 15 | 4 | 1 | 2 | 3 | $10 \times 3+2=32$ |
| 4-bit | 31 | 5 | 1 | 2 | 3 | $8 \times 3=24$ |
| 5-bit | 63 | 6 |  |  |  |  |
| 6-bit | 127 | 7 |  |  |  |  |
| 7-bit | 255 | 8 |  |  |  |  |
| 8-bit | 511 | 9 |  |  |  |  |
| 9-bit | 1,023 | 10 |  |  |  |  |
| 10-bit | 2,047 | 11 |  |  |  |  |
| 16-bit |  |  |  |  |  |  |

b) Consider the SOP Boolean formula for the carry-out $\left(c_{i+1}\right)$ of a 10-bit adder:


Give an example of each of the following:
i) a product term with only two terms
ii) a product term with 11 terms
3. Suppose we have a register file with the following specifications (see Memory Supplement in Lecture 8 on the eLearning system and in your CopyWorks course packet):

- 128 registers numbered from 0 to 127
- each register has 32-bits
- one write port
- two read ports
a) What how many bits(/"wires") would be need for each of the following?
- data to be written for a write port
- specifying the register number of a read port
- specifying the register number of a write port
- output read from a read port
b) How many write enable wires would be needed for the whole register file?
c) How many decoders would be needed in the implementation of the whole register file? Explain how you arrived at that number and specify the type of decoders (i.e., \# of inputs and \# of outputs for each decoder)
d) How many MUXs would be needed in the implementation of the whole register file? Explain how you arrived at that number and specify the type of MUXs (i.e., \# of inputs and \# of outputs for each MUX)

4. Complete the below diagram of a 4-bit register so that it is able to perform the following operations:

- parallel read/output of all bits (just look at the Q values)

Control the MUXs using the following codes:
$00_{2}$ - parallel write/load/input of all bits
$01_{2}$ - logical shift right one bit positions (bits shifted out of least-significant bit are lost and a " 0 " is shifted into the most-significant bit)
$10_{2}$ - arithmetic shift right (sign-extend the most-significant bit)
$11_{2}$ - circular shift left two bit positions (bit shifted out of most-significant bit is shifted into the least-significant bit)
Note: For each D-flip flop, the output of a MUX is used as the D-input as shown below.


Data to Read in Parallel

