1. So far, we have only looked at simple sorts. Recall that all simple sorts consist of nested loops:

- an outer loop that keeps track of the dividing line between the sorted and unsorted part
  - an inner loop that grows the size of the sorted part by one

Usually, the number of inner loop iterations is something like (n-1) + (n-2) + (n-3) + ... + 3 + 2 + 1 = n*(n-1)/2 which is $\Theta(n^2)$. Consider using a heap to sort a list/array.

HEAP METHOD	WHAT IT DOES
`H.add(item)`	Inserts **`item`** in its proper place in the heap, increasing the heap's size by one.
`H.pop()`	*Precondition*: the heap is not empty. Removes and returns the topmost item in the heap.
`H.peek()`	*Precondition*: the heap is not empty. Returns the item at the top of the heap.
`H.isEmpty()`	Returns **`True`** if the heap is empty, or **`False`** otherwise.
`H.__len__()`	Same as **`len(H)`**. Returns the number of items in the heap.
`H.__iter__()`	Same as **`iter(H)`**. Returns an iterator on the heap. This iterator visits the items from the minimum to the maximum.
`H.__str__()`	Same as **`str(H)`**. Returns a string representation of the heap that shows its shape.

[TABLE 18.8] The methods in the heap interface

a) If we put all of the list/array elements into a min-heap, what would we easily be able to determine?

b) What is the $O(\ )$ for buiding the heap with all n items from the array?

c) Write an algorithm to perform the "heap" sort of an array.

d) What is the $O(\ )$ for your algorithm?

2. Another way to do better than the simple sorts is to employ divide-and-conquer to develop more "advanced" sorting algorithms that are recursive: Merge sort and Quick Sort.

In general, a problem can be solved recursively if it can be broken down into smaller problems that are identical in structure to the original problem.

a) What determines the "size" of a sorting problem?

b) How might we break the original problem down into smaller problems that are identical? Are there any additional parameters that might be needed? (recursive algorithms often need extra parameters)

c) What base case(s) (i.e., trival, non-recursive case(s)) might we encounter with recursive sorts?

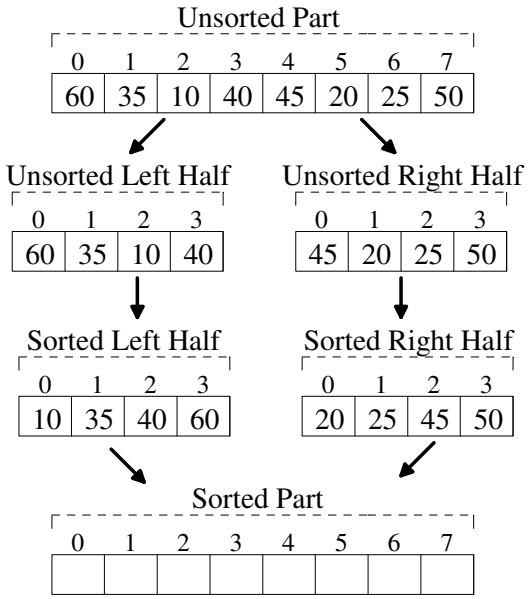
d) Consider why a recursive sort might be more efficient. Assume that I had a simple n^2 sorting algorithm with n = 100, then there is roughly 100^2 / 2 or 5,000 amount of work. Suppose I split the problem down into two smaller problems of size 50.

- If I run the n^2 algorithm on both smaller problems of size 50, then what would be the approximate amount of work?

- If I further solve the problems of size 50 by splitting each of them into two problems of size 25, then what would be the approximate amount of work?

4. The general idea merge sort is as follows. Assume "n" items to sort.

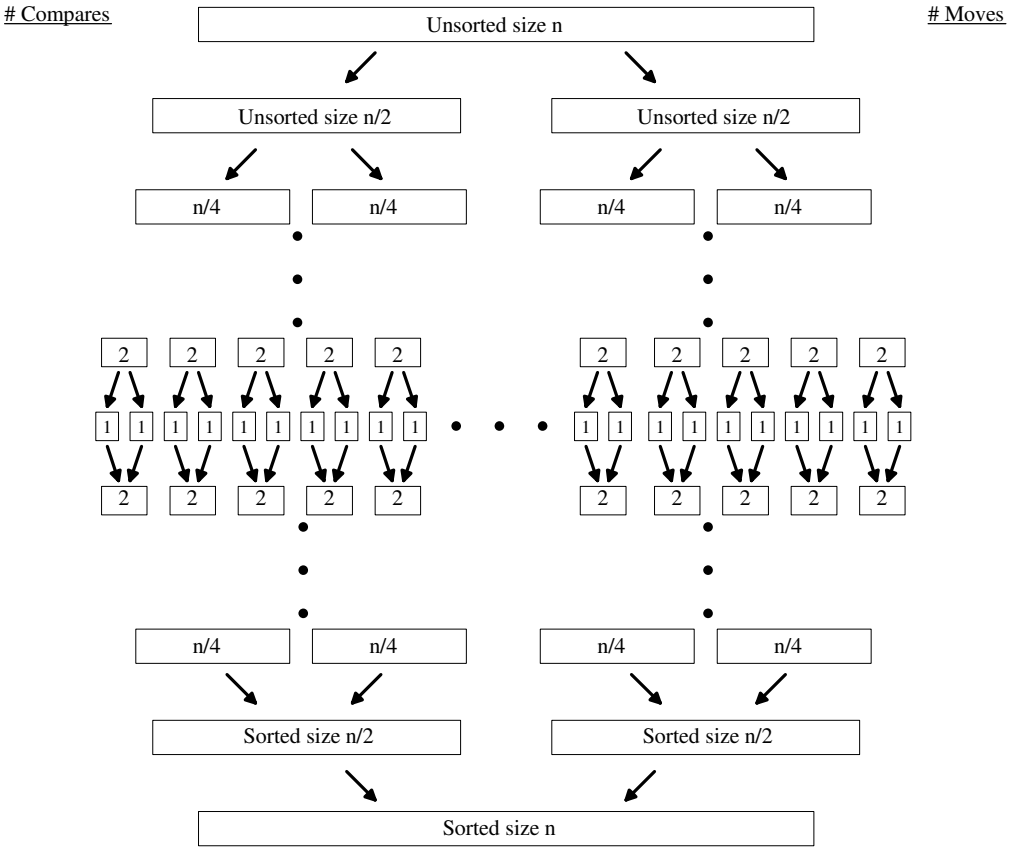
- Split the unsorted part in half to get two smaller sorting problems of about equal size = n/2
- Solve both smaller problem recursively using merge sort
- "Merge" the solution to the smaller problems together to solve the original sorting problem of size n



a) Fill in the sorted part in the above

b) Describe how you filled in the sorted part in the above example?

5. Merge sort is substantially faster than the simple sorts. Let's analyze the number of comparisons and moves of merge sort. Assume "n" items to sort.



a) On each level of the above diagram write the WORST-CASE total number of comparison for that level on the left, and the total number of moves for that level on the right.

b) What is the WORST-CASE total number of comparisons for the whole algorithm (i.e., add all levels)?

c) What is the total number of moves for the whole algorithm (i.e., add all levels)?

d) What is BEST-CASE total number of comparisons for the whole algorithm?

6. *Quick sort* is another advanced sort that often is quicker than merge sort (hence its name). The general idea is as follows. Assume "n" items to sort.

- Select a "random" item in the unsorted part as the *pivot*
- Rearrange (called *partitioning*) the unsorted items such that:

Pivot Index

All items < to Pivot	Pivot Item	All items >= to Pivot

- Quick sort the unsorted part to the left of the pivot
- Quick sort the unsorted part to the right of the pivot

a) What base case(s) would we have?

b) Because of the recursive nature of quick sort, what "extra" parameters would we need to specify the part of the list to sort?