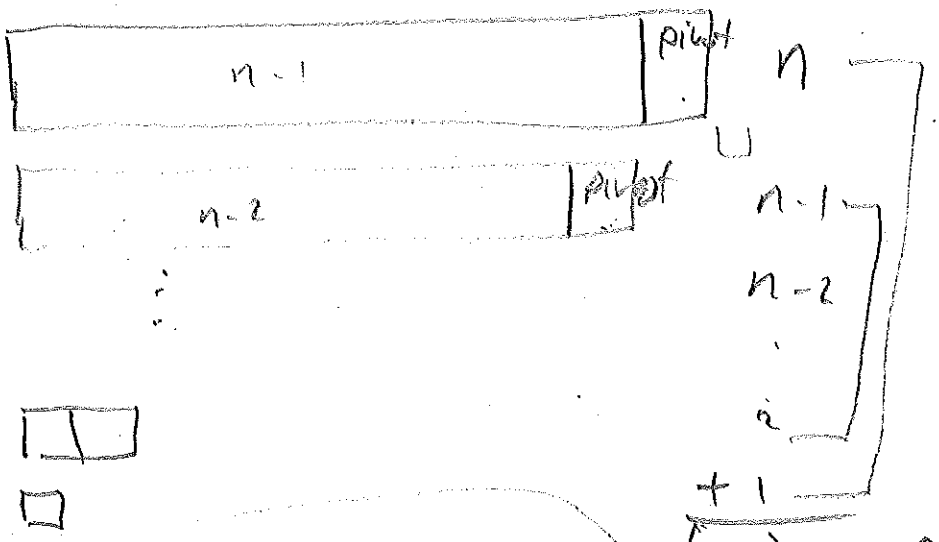
e) Ideally, the pivot item splits the list into two equal size problems. What would be the big-oh for Quick Sort in the best case?

worst case



$(n+1)\frac{n}{2}$ $O(n^2)$

f) What would be the big-oh for Quick Sort in the average case? $O(n \log_2 n)$

Two worst-case pivot values (either end), but anything close to the middle splits list roughly in half.

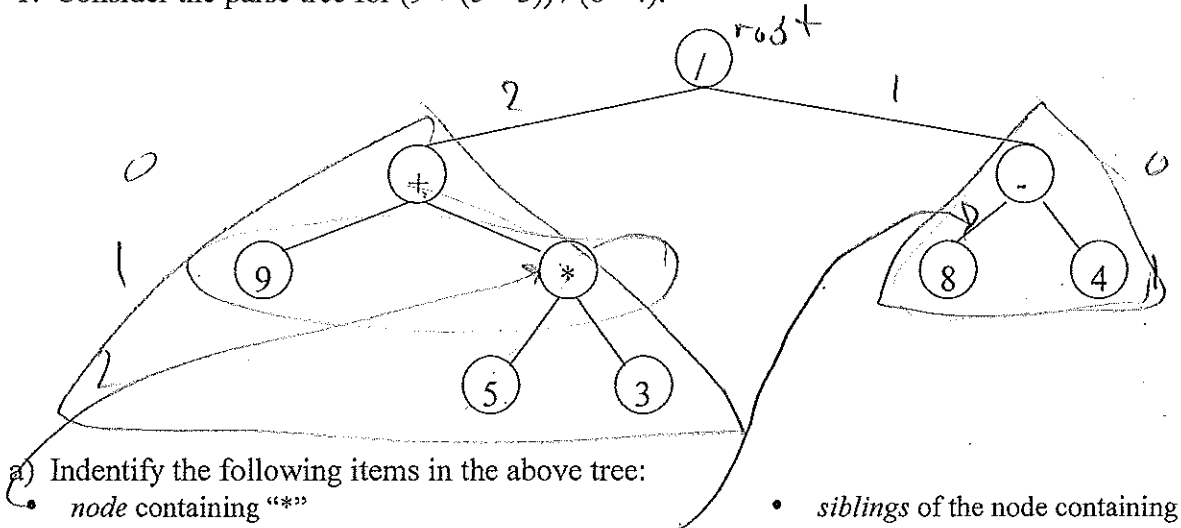
g) The textbook's `partition` code (Listing 5.15 on page 225) selects the first item in the list as the pivot item. However, the above `partition` code selects the middle item of the list to be the pivot. What advantage does selecting the middle item as the pivot have over selecting the first item as the pivot?

If the list was already sorted, then selecting middle item leads to best case.

The textbook's method on a sorted list leads to the worst-case $O(n^2)$ performance.

Data Structures (2) (1) (4) (3) Lecture 18 Name:_____

1. Consider the parse tree for (9 + (5 * 3)) / (8 - 4):



a) Indentify the following items in the above tree:

- *node* containing "*"
- *edge* from node containing "-" to node containing "8"
- *root* node
- *children* of the node containing "+"
- *parent* of the node containing "3"
- *siblings* of the node containing "*"
- *leaf* nodes of the tree
- *subtree* who's root is node contains "+"
- *path* from node containing "+" to node containing "5"
- *branch* from root node to "3"

b) Mark the *levels* of the tree (level is the number of edges on the path from the root)

c) What is the *height* (max. level) of the tree? 3

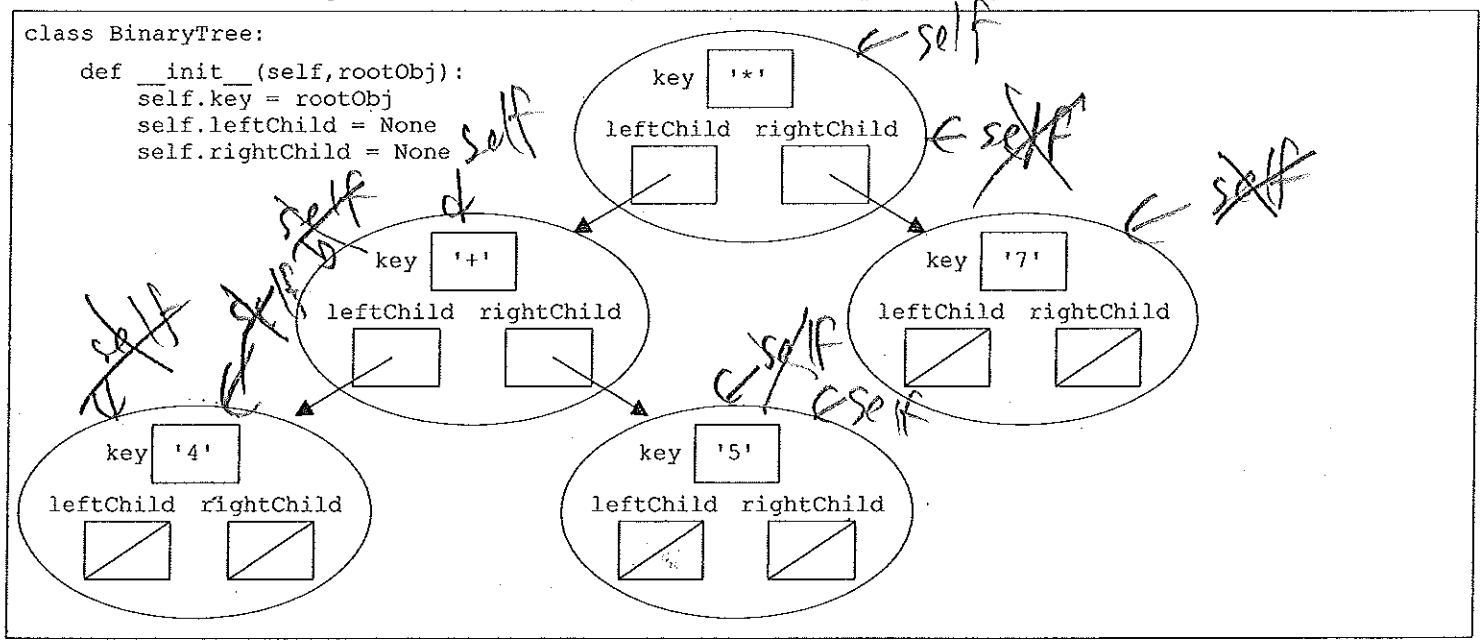
2. In Python an easy way to implement a tree is as a list of lists where a tree look like:

["node value", remaining items are subtrees for the node each implemented as a list of lists]

Complete the list-of-lists representation look like for the above parse tree:

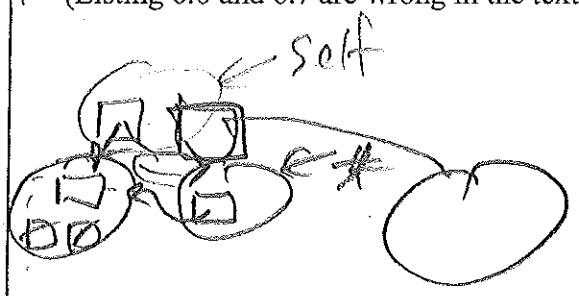
['/', ['+', '9', ['*', 5, 3]], ['-', '8', '4']]

3. Consider a "linked" representations of a BinaryTree. For the expression ((4 + 5) * 7), the binary tree would be:



```
import operator
class BinaryTree:
    def __init__(self,rootObj):
        self.key = rootObj
        self.leftChild = None
        self.rightChild = None

    def insertLeft(self,newNode):
        if self.leftChild == None:
            self.leftChild=BinaryTree(newNode)
        else:
            t = BinaryTree(newNode)
            t.leftChild = self.leftChild
            self.leftChild = t

    def insertRight(self,newNode):
        if self.rightChild == None:
            self.rightChild=BinaryTree(newNode)
        else:
            t = BinaryTree(newNode)
            t.rightChild = self.rightChild
            self.rightChild = t

    def isLeaf(self):
        return ((not self.leftChild) and
                (not self.rightChild))

    def getRightChild(self):
        return self.rightChild

    def getLeftChild(self):
        return self.leftChild

    def setRootVal(self,obj):
        self.key = obj

    def getRootVal(self,):
        return self.key

    def inorder(self):
        if self.leftChild:
            self.leftChild.inorder()
        print(self.key)
        if self.rightChild:
            self.rightChild.inorder()

    def postorder(self):
        if self.leftChild:
            self.leftChild.postorder()
        if self.rightChild:
            self.rightChild.postorder()
        print(self.key)
```

a) Fix the `insertLeft` and `insertRight` code:
(Listing 6.6 and 6.7 are wrong in the text on pp. 242-3)




```
def preorder(self):
    print(self.key)
    if self.leftChild:
        self.leftChild.preorder()
    if self.rightChild:
        self.rightChild.preorder()


def printexp(self):
    if self.leftChild:
        print('(', end=' ')
        self.leftChild.printexp()
    print(self.key, end=' ')
    if self.rightChild:
        self.rightChild.printexp()
        print(')', end=' ')

def postordereval(self):
    opers = {'+':operator.add, '-':operator.sub,
             '*':operator.mul, '/':operator.truediv}
    res1 = None
    res2 = None
    if self.leftChild:
        res1 = self.leftChild.postordereval()
    if self.rightChild:
        res2 = self.rightChild.postordereval()
    if res1 and res2:
        return opers[self.key](res1,res2)
    else:
        return self.key
```

Some corresponding external (non-class) functions:

```
def inorder(tree):
    if tree != None:
        inorder(tree.getLeftChild())
        print(tree.getRootVal())
        inorder(tree.getRightChild())

def printexp(tree):
    if tree.leftChild:
        print('(', end=' ')
        printexp(tree.getLeftChild())
    print(tree.getRootVal(), end=' ')
    if tree.rightChild:
        printexp(tree.getRightChild())
        print(')', end=' ')

def height(tree):
    if tree == None:
        return -1
    else:
        return 1 +
            max(height(tree.leftChild),
                height(tree.rightChild))
```

```
def printexp(tree):
    sVal = ""
    if tree:
        sVal = '(' + printexp(tree.getLeftChild())
        sVal = sVal + str(tree.getRootVal())
        sVal = sVal + printexp(tree.getRightChild()) + ')'
    return sVal

def postordereval(tree):
    opers = {'+':operator.add, '-':operator.sub,
             '*':operator.mul, '/':operator.truediv}
    res1 = None
    res2 = None
    if tree:
        res1 = postordereval(tree.getLeftChild())
        res2 = postordereval(tree.getRightChild())
        if res1 and res2:
            return opers[tree.getRootVal()](res1,res2)
        else:
            return tree.getRootVal()
```

b) If `myTree` is the `BinaryTree` object for the expression: ((4 + 5) * 7), what gets printed by a calls to:

`myTree.inorder()`	`myTree.preorder()`	`myTree.postorder()`	`inorder(myTree)`
4 + 5 * 7	* + 4 5 7	4 5 + 7 *	

c) If `myTree` is the `BinaryTree` object for the expression: ((4 + 5) * 7), what gets printed by a call to `myTree.printexp()`?

d) If `myTree` is the `BinaryTree` object for the expression: ((4 + 5) * 7), what gets printed by a call to `myTree.postordereval()`?

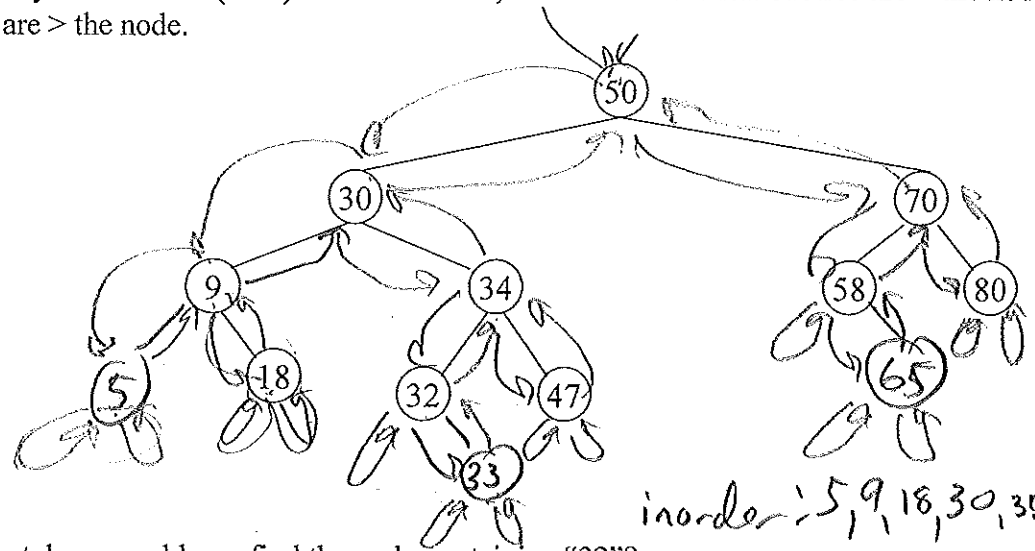
e) From an class/Abstract Data Type (ADT) point of view, why are the external versions of the methods "better"?

The postordereval method only works if the BinaryTree is storing a ParseTree, so the creator of BinaryTree might not have known.

f) Write the `height` method for the `BinaryTree` class.

```
def height(self):
    if self.isLeaf():
        return 0
    return 1 + max(self.leftChild.height(), self.rightChild.height())
```

4. Consider the Binary Search Tree (BST). For each node, all values in the left-subtree are < the node and all values in the right-subtree are > the node.



inorder: 5, 9, 18, 30, 32, 33, 34, 47, 50, 58, 65, 70, 80

a. Starting at the root, how would you find the node containing "32"?
b. Starting at the root, when would you discover that "65" is not in the BST?
c. Starting at the root, where would be the "easiest" place to add "65"?
d. Where would we add "5" and "33"?

preorder: 50, 30, 9, 5, 18, 34, 32, 33, 47, 70, 58, 65, 80

postorder: 5, 18, 9, 33, 32, 47, 34, 30, 65, 58, 80, 70, 50