5. *Quick sort* general idea is as follows.

- Select a "random" item in the unsorted part as the *pivot*
- Rearrange (*partitioning*) the unsorted items such that:
- Quick sort the unsorted part to the left of the pivot
- Quick sort the unsorted part to the right of the pivot

left $3\frac{n}{2} = 1.5n$ Pivot Index right

All items < to Pivot	Pivot Item	All items >= to Pivot

a

a) Given the following `partition` function which returns the index of the pivot after this rearrangement, complete the recursive `quicksortHelper` function.

```
def partition(lyst, left, right):
    # Find the pivot and exchange it with the last item
    middle = (left + right) // 2
    pivot = lyst[middle]
    lyst[middle] = lyst[right]
    lyst[right] = pivot
    # Set boundary point to first position
    boundary = left
    # Move items less than pivot to the left
    for index in range(left, right):
        if lyst[index] < pivot:
            temp = lyst[index]
            lyst[index] = lyst[boundary]
            lyst[boundary] = temp
            boundary += 1
    # Exchange the pivot item and the boundary item
    temp = lyst[boundary]
    lyst[boundary] = lyst[right]
    lyst[right] = temp
    return boundary
```

```
def quicksort(lyst):

    quicksortHelper(lyst, 0, len(lyst) - 1)

def quicksortHelper(lyst, left, right):
    if left<right:
        pivotIndex = partition(lyst,left,right)
        quicksortHelper(lyst, left, pivotIndex-1)
        quicksortHelper(list, pivotIndex+1, right)
```

b) For the list below, trace the first call to partition and determine the resulting list, and value returned.

	0	1	2	3	4	5	6	7	8	left	right	index	boundary	pivot
	26	~~54~~ 17	20	31 54	~~93~~ 44	54	93		44					
lyst:	~~54~~	26	~~93~~	~~17~~	~~50~~	~~31~~	~~44~~	55	~~20~~	0	8			50

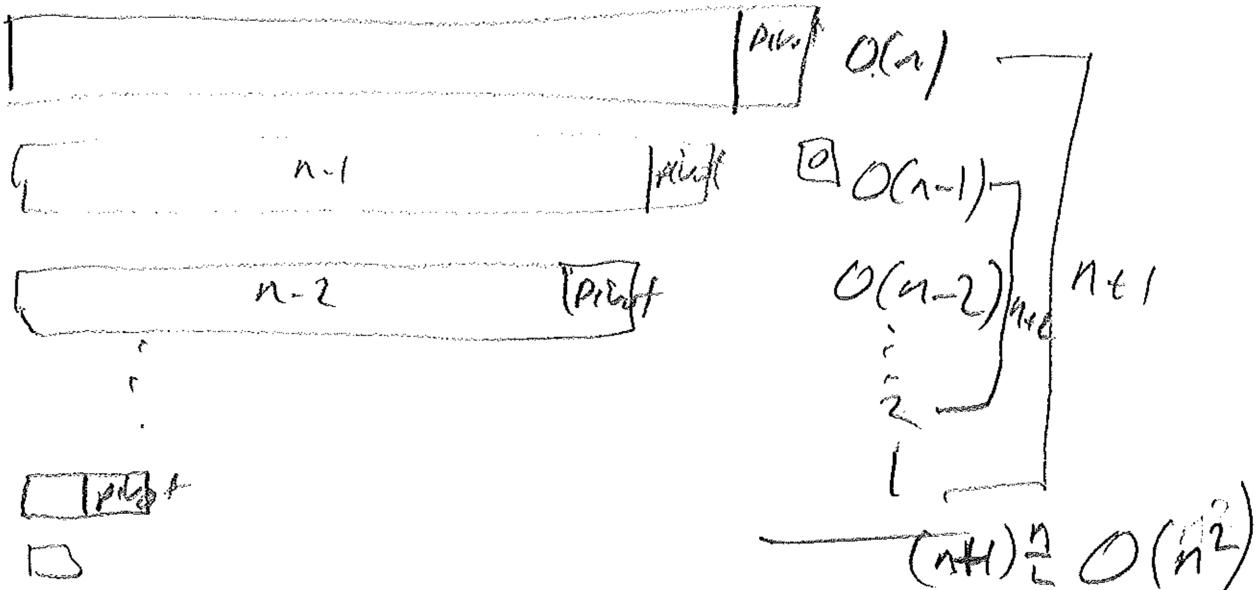
b) What initial arrangement of the list would cause partition to perform the most amount of work?

higher value at middle

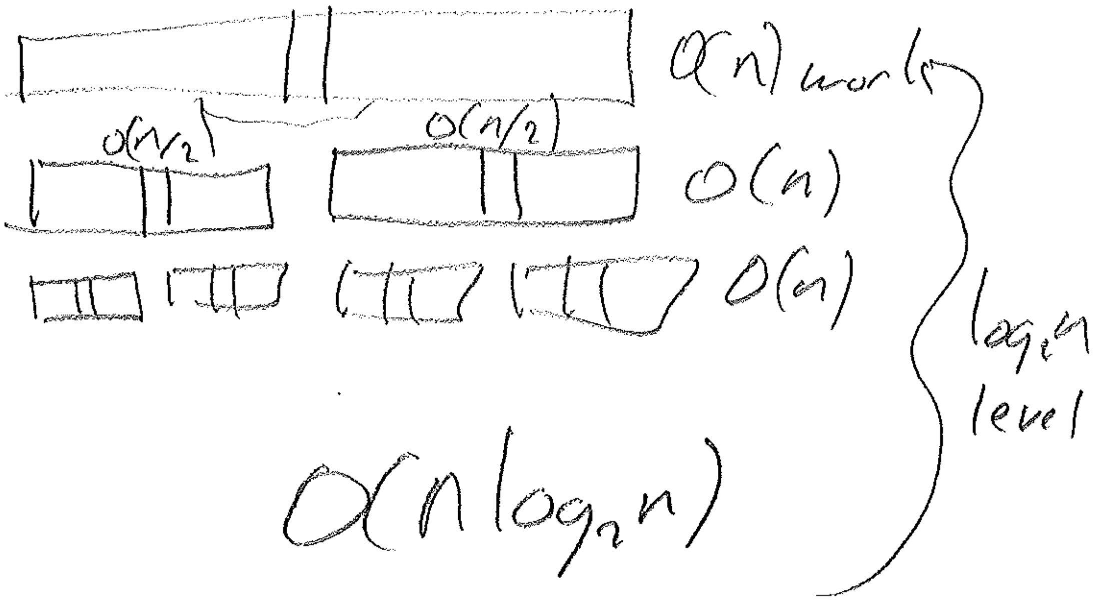
c) Let "n" be the number of items between left and right. What is the worst-case *O*( ) for partition?

O(n)

d) What would be the overall, worst-case $O(\ )$ for Quick Sort?



e) Ideally, the pivot item splits the list into two equal size problems. What would be the big-oh for Quick Sort in the best case?



$O(n \log_2 n)$

f) What would be the big-oh for Quick Sort in the average case?

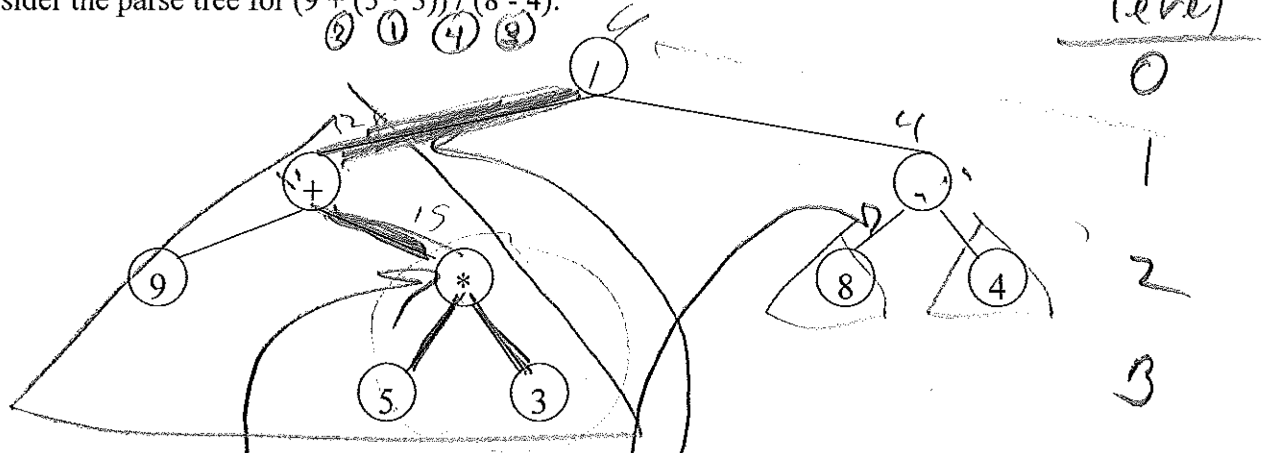
$O(n \log_2 n)$

g) The textbook's `partition` code (Listing 5.15 on page 225) selects the first item in the list as the pivot item. However, the above `partition` code selects the middle item of the list to be the pivot. What advantage does selecting the middle item as the pivot have over selecting the first item as the pivot?

In textbook's partition, a list initial in ascending order would lead to a worst-case $O(n^2)$ performance.

1. Consider the parse tree for (9 + (5 * 3)) / (8 - 4):

② ① ④ ③



a) Indentify the following items in the above tree:

- *node* containing "*"
- *edge* from node containing "-" to node containing "8"
- *root* node (/)
- *children* of the node containing "+" 9, *
- *parent* of the node containing "3" (*)
- *siblings* of the node containing "*" (9)
- *leaf* nodes of the tree 9 - 4
- *subtree* who's root is node contains "+"
- *path* from node containing "+" to node containing "5" +, *, 5
- *branch* from root node to "3"

b) Mark the *levels* of the tree (level is the number of edges on the path from the root)

c) What is the *height* (max. level) of the tree? 3

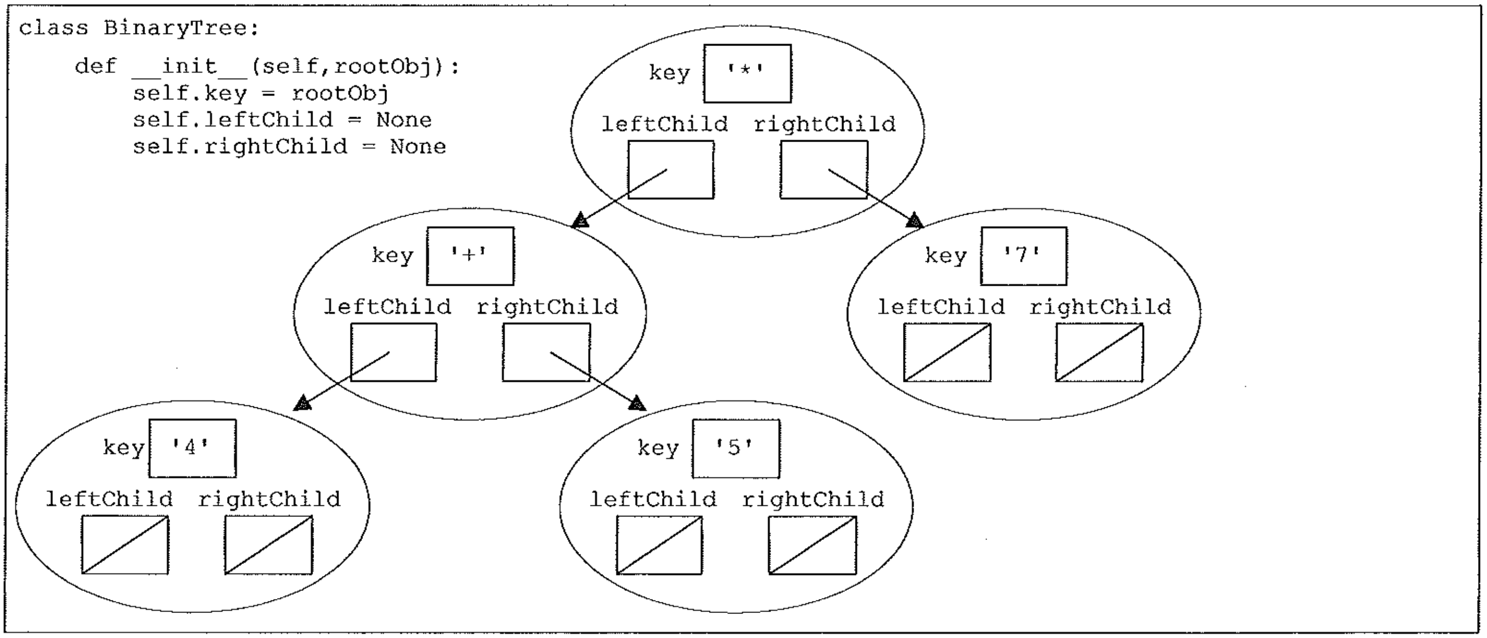
2. In Python an easy way to implement a tree is as a list of lists where a tree look like:

["node value", remaining items are subtrees for the node each implemented as a list of lists]

Complete the list-of-lists representation look like for the above parse tree:

['/', ['+', ['9',[]], ['*',['5',[]], ['3',[]]]], ['-', ['8', [ ]] , ['4', [ ]]]]

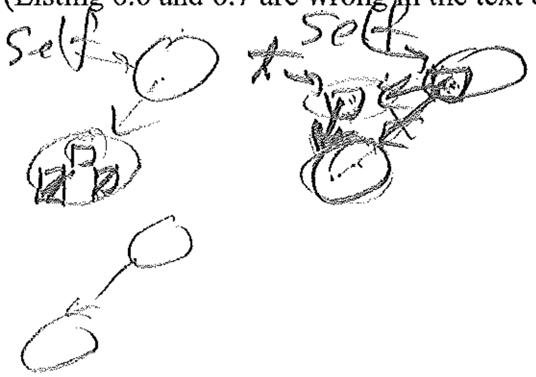
3. Consider a "linked" representations of a BinaryTree. For the expression ((4 + 5) * 7), the binary tree would be:



```
class BinaryTree:
    def __init__(self,rootObj):
        self.key = rootObj
        self.leftChild = None
        self.rightChild = None
```

```
import operator
class BinaryTree:
    def __init__(self,rootObj):
        self.key = rootObj
        self.leftChild = None
        self.rightChild = None

    def insertLeft(self,newNode):
        if self.leftChild == None:
            self.leftChild=BinaryTree(newNode)
        else:
            t = BinaryTree(newNode)
            t.left = self.leftChild
            self.leftChild = t

    def insertRight(self,newNode):
        if self.rightChild == None:
            self.rightChild=BinaryTree(newNode)
        else:
            t = BinaryTree(newNode)
            t.right = self.rightChild
            self.rightChild = t

    def isLeaf(self):
        return ((not self.leftChild) and
                (not self.rightChild))

    def getRightChild(self):
        return self.rightChild

    def getLeftChild(self):
        return self.leftChild

    def setRootVal(self,obj):
        self.key = obj

    def getRootVal(self,):
        return self.key

    def inorder(self):
        if self.leftChild:
            self.leftChild.inorder()
        print(self.key)
        if self.rightChild:
            self.rightChild.inorder()

    def postorder(self):
        if self.leftChild:
            self.leftChild.postorder()
        if self.rightChild:
            self.rightChild.postorder()
        print(self.key)
```

a) Fix the `insertLeft` and `insertRight` code:
(Listing 6.6 and 6.7 are wrong in the text on pp. 242-3)



```
def preorder(self):
    print(self.key)
    if self.leftChild:
        self.leftChild:preorder()
    if self.rightChild:
        self.rightChild.preorder()

def printexp(self):
    if self.leftChild:
        print('(', end=' ')
        self.leftChild.printexp()
    print(self.key, end=' ')
    if self.rightChild:
        self.rightChild.printexp()
        print(')', end=' ')
def postordereval(self):
    opers = {'+':operator.add, '-':operator.sub,
             '*':operator.mul, '/':operator.truediv}
    res1 = None
    res2 = None
    if self.leftChild:
        res1 = self.leftChild.postordereval()
    if self.rightChild:
        res2 = self.rightChild.postordereval()
    if res1 and res2:
        return opers[self.key](res1,res2)
    else:
        return self.key
```

Some corresponding external (non-class) functions:

```
def inorder(tree):
    if tree != None:
        inorder(tree.getLeftChild())
        print(tree.getRootVal())
        inorder(tree.getRightChild())

def printexp(tree):
    if tree.leftChild:
        print('(', end=' ')
        printexp(tree.getLeftChild())
    print(tree.getRootVal(), end=' ')
    if tree.rightChild:
        printexp(tree.getRightChild())
        print(')', end=' ')

def height(tree):
    if tree == None:
        return -1
    else:
        return 1 +
           max(height(tree.leftChild),
               height(tree.rightChild))
```

```
def printexp(tree):
    sVal = ""
    if tree:
        sVal = '(' + printexp(tree.getLeftChild())
        sVal = sVal + str(tree.getRootVal())
        sVal = sVal + printexp(tree.getRightChild()) + ')'
    return sVal

def postordereval(tree):
    opers = {'+':operator.add, '-':operator.sub,
             '*':operator.mul, '/':operator.truediv}
    res1 = None
    res2 = None
    if tree:
        res1 = postordereval(tree.getLeftChild())
       res2 = postordereval(tree.getRightChild())
       if res1 and res2:
           return opers[tree.getRootVal()](res1,res2)
       else:
           return tree.getRootVal()
```

b) If myTree is the BinaryTree object for the expression: ((4 + 5) * 7), what gets printed by a calls to:

myTree.inorder()	myTree.preorder()	myTree.postorder()	inorder(myTree)
4 + 5 * 7	* + 4 5 7	4 5 + 7 *	4 + 5 * 7

c) If myTree is the BinaryTree object for the expression: ((4 + 5) * 7), what gets printed by a call to myTree.printexp()?

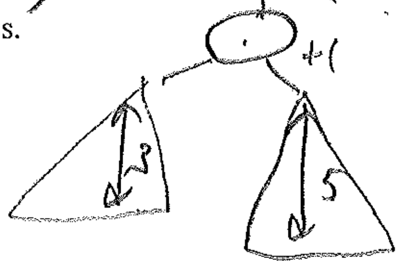
d) If myTree is the BinaryTree object for the expression: ((4 + 5) * 7), what gets returned by a call to myTree.postordereval()? 63

e) From an class/Abstract Data Type (ADT) point of view, why are the external versions of the methods "better"?

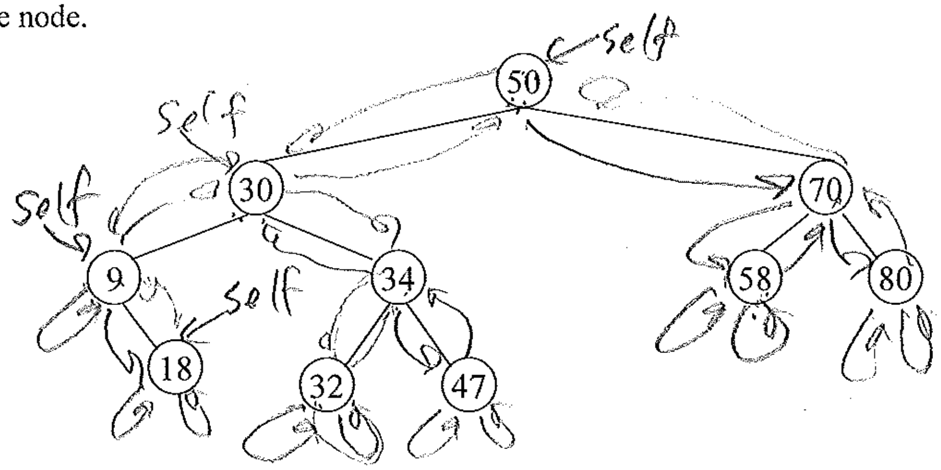
```
htLeft = self.leftChild.height()
htRight = self.rightChild.height()
return 1+max(htLeft, htRight)
```

```
def height( self ):
    if self == None:
        return -1
    1+max(htLeft, htRight)
```

f) Write the height method for the BinaryTree class.



4. Consider the Binary Search Tree (BST). For each node, all values in the left-subtree are < the node and all values in the right-subtree are > the node.



a. Starting at the root, how would you find the node containing "32"?

b. Starting at the root, when would you discover that "65" is not in the BST?

c. What would be the preorder traveral of the BST? 50, 30, 9, 18, 34, 32, 47, 70, 58, 80

d. What would be the postorder traveral of the BST? 18, 9, 32, 47, 34, 30, 58, 80, 70, 50

e. Starting at the root, where would be the "easiest" place to add "65"?

f. Where would we add "5" and "33"?