1. Consider the parse tree for \((9 + (5 \times 3)) / (8 - 4)\):

\[
\begin{array}{c}
/ \\
+ \\
9 \\
- \\
8 \\
5 \\
3 \\
\end{array}
\]

a) Identify the following items in the above tree:
- node containing “\(*\)"
- edge from node containing “-” to node containing “8”
- root node
- children of the node containing “+”
- parent of the node containing “3”

b) Mark the levels of the tree (level is the number of edges on the path from the root)

c) What is the height (max. level) of the tree?

2. In Python an easy way to implement a tree is as a list of lists where a tree look like:

```python
[ "node value", remaining items are subtrees for the node each implemented as a list of lists]
```

Complete the list-of-lists representation look like for the above parse tree:

```python
['/', ['+', ], ['-'], ]
```

3. Consider a “linked” representations of a Binary Tree. For the expression \(((4 + 5) \times 7)\), the binary tree would be:
import operator

class BinaryTree:
    def __init__(self, rootObj):
        self.key = rootObj
        self.leftChild = None
        self.rightChild = None

    def insertLeft(self, newNode):
        if self.leftChild == None:
            self.leftChild = BinaryTree(newNode)
        else:
            t = BinaryTree(newNode)
            t.left = self.leftChild
            self.leftChild = t

    def insertRight(self, newNode):
        if self.rightChild == None:
            self.rightChild = BinaryTree(newNode)
        else:
            t = BinaryTree(newNode)
            t.right = self.rightChild
            self.rightChild = t

    def isLeaf(self):
        return ((not self.leftChild) and
                (not self.rightChild))

    def getRightChild(self):
        return self.rightChild

    def getLeftChild(self):
        return self.leftChild

    def setRootVal(self, obj):
        self.key = obj

    def getRootVal(self):
        return self.key

    def inorder(self):
        if self.leftChild:
            self.leftChild.inorder()
        print(self.key)
        if self.rightChild:
            self.rightChild.inorder()

    def postorder(self):
        if self.leftChild:
            self.leftChild.postorder()
        if self.rightChild:
            self.rightChild.postorder()
        print(self.key)

    def preorder(self):
        print(self.key)
        if self.leftChild:
            self.leftChild.preorder()
        if self.rightChild:
            self.rightChild.preorder()

    def printexp(self):
        if self.leftChild:
            print('(', end=' ')
            self.leftChild.printexp()
        print(self.key, end=' ')
        if self.rightChild:
            self.rightChild.printexp()
            print(')', end=' ')

    def postordereval(self):
        oper = {'+': operator.add, '-': operator.sub,
                '*': operator.mul, '/': operator.truediv}
        res1 = None
        res2 = None
        if self.leftChild:
            res1 = self.leftChild.postordereval()
        if self.rightChild:
            res2 = self.rightChild.postordereval()
        if res1 and res2:
            return oper[self.key](res1, res2)
        else:
            return self.key

some corresponding external (non-class) functions:

def inorder(tree):
    if tree != None:
        inorder(tree.getLeftChild())
    print(tree.getRootVal())
    inorder(tree.getRightChild())

def printexp(tree):
    sVal = ""
    if tree:
        sVal = "(' + printexp(tree.getLeftChild())
        sVal = sVal + str(tree.getRootVal())
        sVal = sVal + printexp(tree.getRightChild()) + ')'
    return sVal

def postordereval(tree):
    oper = {'+': operator.add, '-' : operator.sub,
            '*': operator.mul, '/': operator.truediv}
    res1 = None
    res2 = None
    if tree:
        res1 = postordereval(tree.getLeftChild())
        res2 = postordereval(tree.getRightChild())
    if res1 and res2:
        return oper[tree.getRootVal()](res1, res2)
    else:
        return tree.getRootVal()
b) If `myTree` is the `BinaryTree` object for the expression: `(4 + 5) * 7`, what gets printed by a call to:

<table>
<thead>
<tr>
<th>method</th>
<th>result</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>myTree.inorder()</code></td>
<td></td>
</tr>
<tr>
<td><code>myTree.preorder()</code></td>
<td></td>
</tr>
<tr>
<td><code>myTree.postorder()</code></td>
<td></td>
</tr>
<tr>
<td><code>inorder(myTree)</code></td>
<td></td>
</tr>
</tbody>
</table>

c) If `myTree` is the `BinaryTree` object for the expression: `(4 + 5) * 7`, what gets printed by a call to `myTree.printexp()`?

d) If `myTree` is the `BinaryTree` object for the expression: `(4 + 5) * 7`, what gets returned by a call to `myTree.postordereval()`?

e) From a class/Abstract Data Type (ADT) point of view, why are the external versions of the methods “better”?

f) Write the `height` method for the `BinaryTree` class.

4. Consider the Binary Search Tree (BST). For each node, all values in the left-subtree are < the node and all values in the right-subtree are > the node.

```
      50
     /  \
   30   70
  /  \
9   34  58
/  \
18  32  47
    /
   18
```

a. Starting at the root, how would you find the node containing “32”?
b. Starting at the root, when would you discover that “65” is not in the BST?
c. What would be the preorder traversal of the BST?
d. What would be the postorder traversal of the BST?
e. Starting at the root, where would be the “easiest” place to add “65”?
f. Where would we add “5” and “33”?