

4. Section 7.5 uses recursion and the run-time stack to implement a DFS traversal. The DFSGraph uses a time attribute to note when a vertex is first encountered (discovery attribute) in the depth-first search and when a vertex is backtracked through (finish attribute). Consider the graph for making pancakes where vertices are steps and edges represent the partial order among the steps.

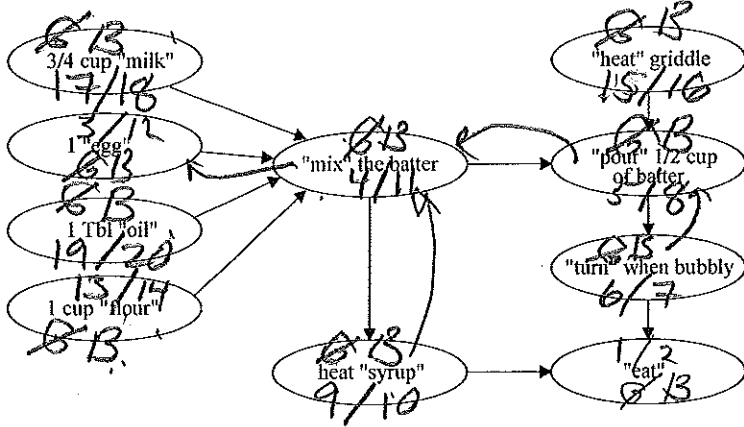
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from graph import Graph
class DFSGraph(Graph):

    def __init__(self):
        super().__init__()
        self.time = 0

    def dfs(self):
        for aVertex in self:
            aVertex.setColor('white')
            aVertex.setPred(-1)
        for aVertex in self:
            if aVertex.getColor() == 'white':
                self.dfsvisit(aVertex)

    def dfsvisit(self, startVertex):
        startVertex.setColor('gray')
        self.time += 1
        startVertex.setDiscovery(self.time)
        for nextVertex in startVertex.getConnections():
            if nextVertex.getColor() == 'white':
                nextVertex.setPred(startVertex)
                self.dfsvisit(nextVertex)
        startVertex.setColor('black')
        self.time += 1
        startVertex.setFinish(self.time)
    
```



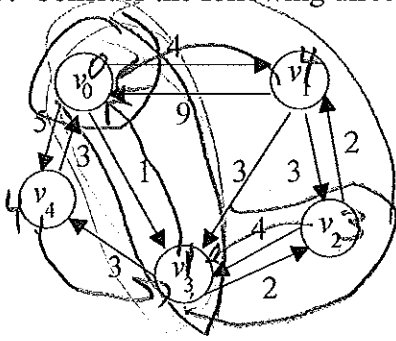
a) Assume (why is this a bad assumption???) that the for-loops always iterate through the vertices alphabetically (e.g., "eat", "egg", "flour", ...) by their id. Write on the above graph the discovery and finish attributes assigned to each vertex by executing the dfs method?

(see above)

b) A topological sort algorithm can use the dfs discovery and finish attributes to determine a proper order to avoid putting the "cart before the horse." For example, we don't want to "pour 1/2 cup of batter" before we "mix the batter", and we don't want to "mix the batter" until all the ingredients have been added. Outline the steps to perform a topological sort.

Descending order of finish times.

5. Consider the following directed graph (diagram).



Dijkstra's Algorithm is a *greedy algorithm* that finds the shortest path from some vertex, say v_0 , to all other vertices. A *greedy algorithm*, unlike divide-and-conquer and dynamic programming algorithms, DOES NOT divide a problem into smaller subproblems. Instead a greedy algorithm builds a solution by making a sequence of choices that look best ("locally" optimal) at the moment without regard for past or future choices (no backtracking to fix bad choices). Dijkstra's algorithm builds a subgraph by repeatedly selecting the next closest vertex to v_0 that is not already in the subgraph. Initially, only vertex v_0 is in the subgraph with a distance of 0 from itself.

a) What would be the order of vertices added to the subgraph during Dijkstra's algorithm?

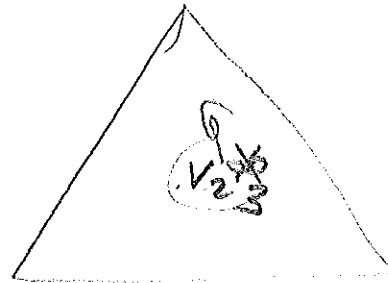
v_0, v_3, v_2, v_1, v_4

b) What *greedy criteria* did you use to select the next vertex to add to the subgraph?

smallest distance to v_0 using only vertices in subgraph

c) What data structure could be used to efficiently determine that selection?

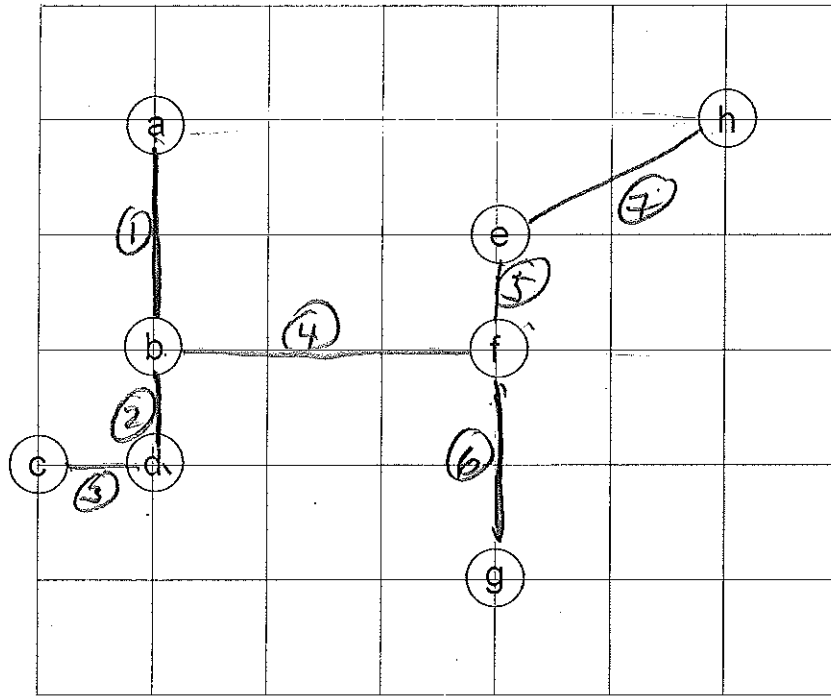
Min Heap $O(\log_2 n)$
 # vertices not in the subgraph



d) How might this data structure need to be modified?

decrease key

1. Suppose you had a map of settlements on the planet X
(Assume edges could connect all vertices with their Euclidean distances as their costs)



We want to build roads that allow us to travel between any pair of cities. Because resources are scarce, we want the total length of all roads built to be minimal. Since all cities will be connected anyway, it does not matter where we start, but assume we start at "a".

- a) Assuming we start at city "a" which city would you connect first? b Why this city?

closest to 'a'

- b) What city would you connect next to expand your partial road network?

close vertex not in partial road system to something in the partial road system. (min/leg)

- c) What would be some characteristics of the resulting "graph" after all the cities are connected?

"connected graph" without cycles \Rightarrow tree
"min. spanning tree," MST

- d) Does your algorithm come up with the overall best (globally optimal) result?

Yes.