1. Section 6.6 discusses a very “non-intuitive”, but powerful list/array-based approach to implement a priority queue, call a binary heap. The list/array is used to store a complete binary tree (a full tree with any additional leaves as far left as possible) with the items being arranged by heap-order property, i.e., each node is \( \leq \) either of its children. An example of a min heap “viewed” on a complete binary tree would be:

```
  6
 / \
 |     |
 |     |
300[8] 125[9]
```

Python List actually used to store heap items:

<table>
<thead>
<tr>
<th>not used</th>
<th>6</th>
<th>15</th>
<th>10</th>
<th>114</th>
<th>20</th>
<th>20</th>
<th>50</th>
<th>300</th>
<th>125</th>
<th>117</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

a) For the above heap, the list/array indexes are indicated in [ ]'s. For a node at index \( i \), what is the index of:
- its left child if it exists:
- its right child if it exists:
- its parent if it exists:

b) What would the above heap look like after inserting 13 and then 3? (show the changes on above tree)

General Idea of insert(newItem):
- append newItem to the end of the list (easy to do, but violates heap-order property)
- restore the heap-order property by repeatedly swapping the newItem with its parent until it percolates to correct spot

c) What is the big-oh notation for inserting a new item in the heap?

d) Complete the code for the percUp method used by insert.

```python
class BinHeap:
    def __init__(self):
        self.heapList = [0]
        self.currentSize = 0
    def percUp(self,currentIndex):
        parentIndex =
        while
            def insert(self, k):
                self.heapList.append(k)
                self.currentSize = self.currentSize + 1
                self.percUp(self.currentSize)
```
2. Now let us consider the `delMin` operation that removes and returns the minimum item.

Python List actually used to store heap items

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<td></td>
</tr>
</tbody>
</table>

a) What item would `delMin` remove and return from the above heap?

b) What is the quickest way to fill the hole left by `delMin`?

c) What new problem does this cause?

General Idea of `delMin()`:
- remember the minimum value so it can be returned later (easy to find - at index 1)
- copy the last item in the list to the root, delete it from the right end, decrement size
- restore the heap-order property by repeatedly swapping this item with its smallest child until it **percolates down** to the correct spot
- return the minimum value

d) What would the above heap look like after `delMin`? (show the changes on above tree)

e) Complete the code for the `percDown` method used by `delMin`.

```python
class BinHeap:
    def minChild(self, i):
        if i * 2 + 1 > self.currentSize:  # if only left child
            return i * 2
        else:
            if self.heapList[i * 2] < self.heapList[i * 2 + 1]:
                return i * 2
            else:
                return i * 2 + 1

def delMin(self):
    retval = self.heapList[1]
    self.currentSize = self.currentSize - 1
    self.heapList.pop()
    self.percDown(1)
    return retval

def percDown(self, currentIndex):
```

f) What is the big-oh notation for `delMin`?
Once we have a working BinHeap, then implementing the PriorityQueue class using a BinHeap is a piece of cake:

```python
from binheap import BinHeap

class PriorityQueue:
    def __init__(self):
        self._heap = BinHeap()

    def isEmpty(self):
        return self._heap.isEmpty()

    def enqueue(self, item):
        self._heap.insert(item)

    def dequeue(self):
        return self._heap.delMin()

    def size(self):
        return self._heap.size()

    def __str__(self):
        return str(self._heap)
```

3. A “list” is a generic term for a sequence of items in a linear arrangement. Unlike stacks, queues and deques access to list items is not limited to either end, but can be from any position in the list. The general terminology of a list is illustrated by:

```
"Abstract view of a list"     [0]  [1]  [2]  [3]  index/position in the list
   'w'  'a'  'y'  'c'

head

```

There are three broad categories of list operations that are possible:

* **index-based operations** - the list is manipulated by specifying an index location, e.g.,
  myList.insert(3, item)  # insert item at index 3 in myList

* **content-based operations** - the list is manipulated by specifying some content (i.e., item value), e.g.,
  myList.remove(item)  # removes the item from the list based on its value

* **cursor-base operations** - a cursor (current position) can be moved around the list, and it is used to identify list items to be manipulated, e.g.,
  myList.first()  # sets the cursor to the head item of the list
  myList.next()  # moves the cursor one position toward the tail of the list
  myList.remove()  # deletes the second item in the list because that’s where the cursor is currently located

The following table summarizes the operations from the three basic categories on a list, L:

<table>
<thead>
<tr>
<th>Index-based operations</th>
<th>Content-based operations</th>
<th>Cursor-based operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>L.insert(index, item)</td>
<td>L.add(item)</td>
<td>L.hasNext()</td>
</tr>
<tr>
<td>item = L[index]</td>
<td>L.remove(item)</td>
<td>L.next()</td>
</tr>
<tr>
<td>L[index] = newValue</td>
<td>L.search(item) #return Boolean</td>
<td>L.hasPrevious()</td>
</tr>
<tr>
<td>L.pop(index)</td>
<td>i = L.index(item)</td>
<td>L.previous()</td>
</tr>
</tbody>
</table>

Built-in Python lists are unordered with a mixture of index-based and content-based operations. We know they are implemented using a contiguous block of memory (i.e., an array). The textbook talks about an unordered list ADT, and a sorted list ADT which is more content-based. Both are implemented using a singly-linked list.

a) Why would a singly-linked list be a bad choice for implementing a cursor-based list ADT?