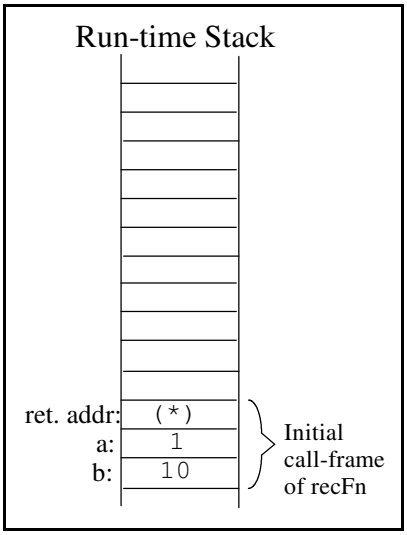
Fall 2017 Name: _____

Data Structures - Test 2

Question 1. (10 points) What is printed by the following program? Output:

```
def recFn(a, b):
    print( a, b )
    if  a == b:
        return b
    elif a > b:
        return a
    else:
        return a + recFn(a + 1, b - 2) - b
                        (**)

print("Result = ", recFn(1, 10))
                        (*)
```



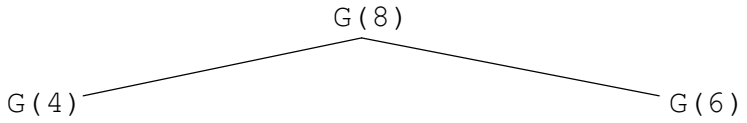
Question 2. a) (12 points) Write a recursive Python function to compute the following mathematical function, G(n):

G(n) = n for all values of n ≤ 3 (e.g., G(2) value is 2)

G(n) = G(n-4) + G(n-2) for all values of n > 3.

```
def G(n):
```

b) (8 points) For the above recursive function G(n), complete the calling-tree for G(8).



c) (3 points) What is the value of G(8)?

d) (2 points) What is the maximum number of call-frames of G on the run-time stack when calculating G(8) recursively?

Question 3. (15 points) Consider the following simple sorts discussed in class -- all of which sort in ascending order.

```
def bubbleSort(myList):
    for lastUnsortedIndex in range(len(myList)-1,0,-1):
        alreadySorted = True
        for testIndex in range(lastUnsortedIndex):
            if myList[testIndex] > myList[testIndex+1]:
                temp = myList[testIndex]
                myList[testIndex] = myList[testIndex+1]
                myList[testIndex+1] = temp
                alreadySorted = False
        if alreadySorted:
            return
```

```
def insertionSort(myList):
    for firstUnsortedIndex in range(1,len(myList)):
        itemToInsert = myList[firstUnsortedIndex]
        testIndex = firstUnsortedIndex - 1
        while testIndex >= 0 and myList[testIndex] > itemToInsert:
            myList[testIndex+1] = myList[testIndex]
            testIndex = testIndex - 1

        myList[testIndex + 1] = itemToInsert
```

```
def selectionSort(aList):
    for lastUnsortedIndex in range(len(aList)-1, 0, -1):
        maxIndex = 0
        for testIndex in range(1, lastUnsortedIndex+1):
            if aList[testIndex] > aList[maxIndex]:
                maxIndex = testIndex
        # exchange the items at maxIndex and lastUnsortedIndex
        temp = aList[lastUnsortedIndex]
        aList[lastUnsortedIndex] = aList[maxIndex]
        aList[maxIndex] = temp
```

Timings of Above Sorting Algorithms on 10,000 items (seconds)			
Type of sorting algorithm	Initial Ordering of Items		
	Descending	Ascending	Random order
bubbleSort.py	24.5	0.002	16.5
insertionSort.py	14.2	0.004	7.3
selectionSort.py	7.3	7.7	6.8

a) Explain why insertionSort on a descending list (14.2 s) takes longer than insertionSort on a random list (7.3 s).

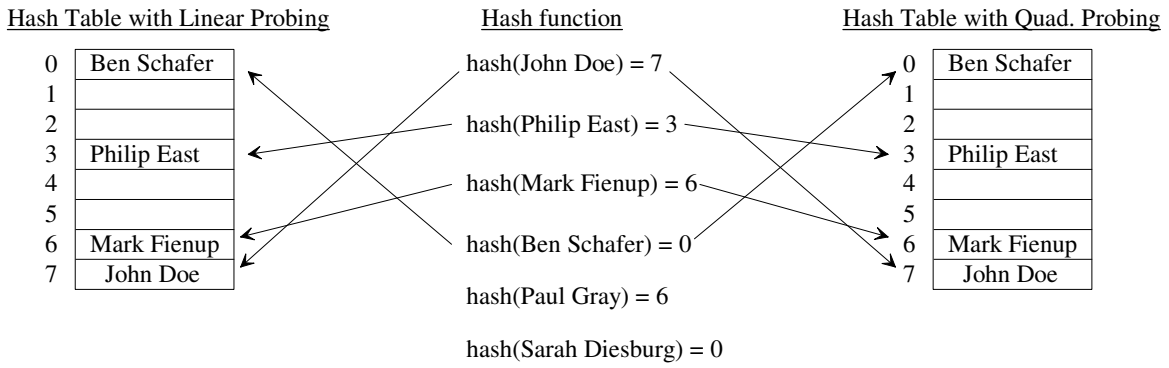
b) Explain why insertionSort on a descending list (14.2 s) takes longer than selectionSort on a descending list (7.3 s).

c) Explain why bubble sort is $O(n^2)$ in the worst-case.

Question 4. Two common rehashing strategies for open-address hashing are linear probing and quadratic probing:

quadratic probing	Check the square of the attempt-number away for an available slot, i.e., [home address + ((rehash attempt #)^2 +(rehash attempt #))//2] % (hash table size), where the hash table size is a power of 2. Integer division is used above

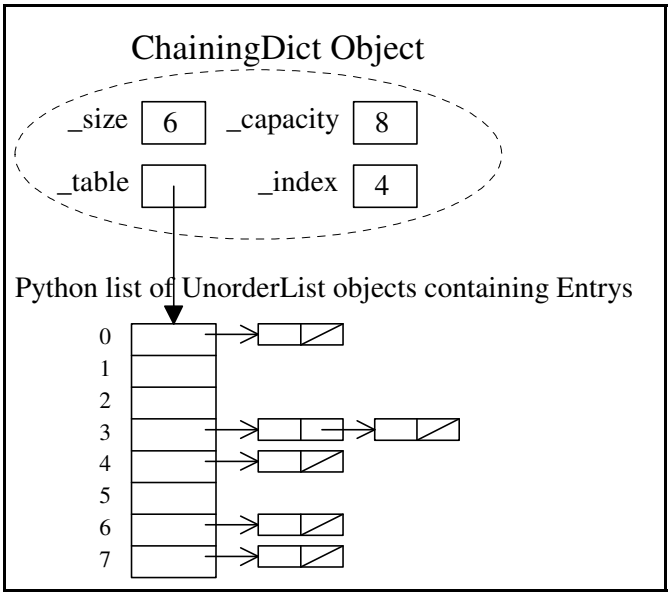
a) (8 points) Insert "Paul Gray" and then "Sarah Diesburg" using Linear (on left) and Quadratic (on right) probing.



b) (4 points) In open-address hashing (like the pictures above), how do we handle deleting items in the hash table?

c) (3 points) In open-address hashing (like the pictures above), how do deleted items effect performance of searching?

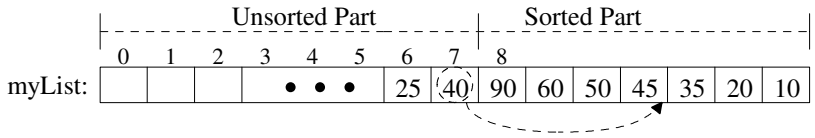
d) (5 points) In closed-address hashing (e.g., ChainingDict like picture below), if the load factor (# items / hash table size) is close to 1, say 0.99, would you expect average-case searches of O(1)? (Justify your answer)



Question 5. (20 points) In class we discussed the insertionSort code shown in question 3 on page 2 which sorts in ascending order (smallest to largest) and builds the sorted part on the left-hand side of the list.

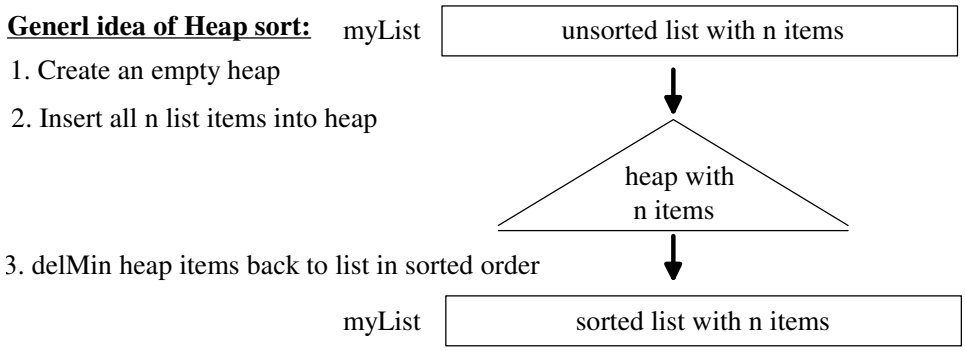
For this question write a variation of insertion sort that:

- sorts in **descending order** (largest to smallest), and
- builds the **sorted part on the right-hand side** of the list, i.e.,




```
def insertionSortVariation(myList):
```

Question 6. Recall the general idea of Heap sort which uses a min-heap (class `BinHeap` with methods: `BinHeap(), insert(item), delMin(), isEmpty(), size()`)) to sort a list.



a) (5 points) Complete the code for `heapSort` so that it **sorts in descending order**

```
from bin_heap import BinHeap
def heapSort(myList):
   myHeap = BinHeap()  # Create an empty heap
```

b) (5 points) Determine the overall *O*( ) for your heap sort and briefly justify your answer. Let n = len(myList).