5. **Quick Sort** general idea is as follows.

- Select a "random" item in the unsorted part as the pivot
- **Rearrange (partitioning)** the unsorted items such that:
  - Quick sort the unsorted part to the left of the pivot
  - Quick sort the unsorted part to the right of the pivot

a) Given the following **partition** function which returns the index of the pivot after this rearrangement, complete the recursive **quicksortHelper** function.

```python
def partition(lyst, left, right):
    # Find the pivot and exchange it with the last item
    middle = (left + right) // 2
    pivot = lyst[middle]
    lyst[middle], lyst[right] = lyst[right], pivot
    # Set boundary point to first position
    boundary = left
    # Move items less than pivot to the left
    for index in range(left, right):
        if lyst[index] < pivot:
            temp = lyst[index]
            lyst[index] = lyst[boundary]
            lyst[boundary] = temp
            boundary += 1
    # Exchange the pivot item and the boundary item
    temp = lyst[boundary]
    lyst[boundary] = lyst[right]
    lyst[right] = temp
    return boundary
```

d) For the list below, trace the first call to partition and determine the resulting list, and value returned.

```
lyst: 54 26 93 17 30 31 44 55 25
```

b) What initial arrangement of the list would cause partition to perform the most amount of work?

c) Let "n" be the number of items between left and right. What is the worst-case $O(\_\_\_\_)$ for partition?

$$O(n)$$
Partition code

1. Pick pivot as middle item and move it to right end to get it out of way.

2. Scan from 1 to n if item found thats < pivot swap it with boundary item.
   Note: boundary keeps track of the dividing line between items < pivot

3. Swap pivot to boundary to complete the partition, and return boundary as pivot index.
d) What would be the overall, worst-case \( O(\cdot) \) for Quick Sort?

\[ n-1 \text{ compares by partition} \]
\[ n-2 \text{ compares} \]
\[ n-3 \]
\[ \vdots \]
\[ +1 \]
\[ n \left( \frac{n-1}{2} \right) \cdot O(n^2) \]


e) Ideally, the pivot item splits the list into two equal size problems. What would be the big-oh for Quick Sort in the best case?

\[ \left\{ \begin{array}{c}
\left( n-1 \right) \text{ compares} \\
2 \left( n/2 \right) \text{ compares} \\
\log_2 n \text{ levels}
\end{array} \right. \]
\[ O(n \log_2 n) \]

f) What would be the big-oh for Quick Sort in the average case?

\[ O(n \log_2 n) \]

g) The textbook’s partition code (Listing 5.15 on page 225) selects the first item in the list as the pivot item. However, the above partition code selects the middle item of the list to be the pivot. What advantage does selecting the middle item as the pivot have over selecting the first item as the pivot?
1. Consider the parse tree for \((9 \div (5 \times 3)) / (8 - 4)\):

![Parse Tree Diagram]

a) Identify the following items in the above tree:
- node containing "\(*\)"
- edge from node containing "-" to node containing "8"
- root node
- children of the node containing "\(*\)"
- parent of the node containing "3"
- siblings of the node containing "\(*\)"
- leaf nodes of the tree 9, 5, 3, 8, 4
- subtree who's root is node containing "\(*\)"
- path from node containing "\(*\)" to node containing "5"
- branch from root node to "3"

b) Mark the levels of the tree (level is the number of edges on the path from the root).

c) What is the height (max. level) of the tree? 3

2. In Python an easy way to implement a tree is as a list of lists where a tree look like:

\[
[\text{"node value"}, \text{remaining items are subtrees for the node each implemented as a list of lists}]
\]

Complete the list-of-lists representation look like for the above parse tree:

\[
[\text{"\(*\)"}, [\text{"\(+\)"}, [\text{"9"}, \text{["\(*\)"}, \text{\([\text{"5"}, \text{\([\text{"8"}, \text{\([\text{"4"}])}\)]}\)]}]\]}
\]

3. Consider a "linked" representations of a BinaryTree. For the expression \(((4 + 5) * 7)\), the binary tree would be:

```python
class BinaryTree:
    def __init__(self, rootObj):  
        self.key = rootObj  
        self.leftChild = None
        self.rightChild = None
```

![Linked Binary Tree Diagram]
Data Structures

**Lecture 18**

Name:

---

```python
import operator
class BinaryTreeNode:
    def __init__(self, rootObj):
        self.key = rootObj
        self.leftChild = None
        self.rightChild = None

    def insertLeft(self, newNode):
        if self.leftChild == None:
            self.leftChild = BinaryTree(newNode)
        else:
            t = BinaryTree(newNode)
            t.left = self.leftChild
            self.leftChild = t

    def insertRight(self, newNode):
        if self.rightChild == None:
            self.rightChild = BinaryTree(newNode)
        else:
            t = BinaryTree(newNode)
            t.right = self.rightChild
            self.rightChild = t

    def isLeaf(self):
        return (not self.leftChild) and (not self.rightChild)

    def getRightChild(self):
        return self.rightChild

    def getLeftChild(self):
        return self.leftChild

    def setRootVal(self, obj):
        self.key = obj

    def getRootVal(self):
        return self.key

    def inOrder(self):
        if self.leftChild:
            self.leftChild.inOrder()
        print(self.key)
        if self.rightChild:
            self.rightChild.inOrder()

    def postOrder(self):
        if self.leftChild:
            self.leftChild.postOrder()
        if self.rightChild:
            self.rightChild.postOrder()
        print(self.key)

---

def inOrder(tree):
    if tree != None:
        inOrder(tree.getLeftChild())
        print(tree.getRootVal())
        inOrder(tree.getRightChild())

def printExp(tree):
    if tree.leftChild:
        print('(', end=' ')
        printExp(tree.leftChild())
        print(tree.getRootVal(), end=' ')
    if tree.rightChild:
        printExp(tree.getRightChild())
        print(')', end='')

def height(tree):
    if tree == None:
        return -1
    else:
        return 1 + max(height(tree.leftChild), height(tree.rightChild))

def preOrder(self):
    print(self.key)
    if self.leftChild:
        self.leftChild.preOrder()
    if self.rightChild:
        self.rightChild.preOrder()

def printExp(self):
    if self.leftChild:
        print('+', end=' ')
        self.leftChild.printExp()
        print(self.key, end=' ')
    if self.rightChild:
        self.rightChild.printExp()
        print(')', end=' ')

def postOrder(self):
    for op in ['+', '-', '*', '/']:
        self.leftChild = self.leftChild.postOrder()
    if self.rightChild:
        res1 = self.leftChild.postOrder()
        res2 = self.rightChild.postOrder()
        return res1 + res2
    return self.key

Some corresponding external (non-class) functions:

```
b) If \texttt{myTree} is the 
\texttt{BinaryTree} object for the expression: \((4 + 5) \times 7\), what gets printed by a calls to:

\begin{center}
\begin{tabular}{|c|c|c|c|}
\hline
\texttt{myTree.inorder()} & \texttt{myTree.preorder()} & \texttt{myTree.postorder()} & \texttt{inorder(myTree)} \\
\hline
4 & | & | & \texttt{null} \\
\hline
+ & | & | & \texttt{null} \\
\hline
5 & | & | & \texttt{null} \\
\hline
\end{tabular}
\end{center}

c) If \texttt{myTree} is the \texttt{BinaryTree} object for the expression: \((4 + 5) \times 7\), what gets printed by a call to \texttt{myTree.printexp()}?

d) If \texttt{myTree} is the \texttt{BinaryTree} object for the expression: \((4 + 5) \times 7\), what gets returned by a call to \texttt{myTree.postordereval()}?

e) Write the \texttt{height} method for the \texttt{BinaryTree} class.

\[
\text{height}(\text{tree}) = \begin{cases} 
1 & \text{if } \text{tree} \text{ is a leaf} \\
\max(\text{height(left child)}, \text{height(right child)}) + 1 & \text{otherwise}
\end{cases}
\]

4. Consider the Binary Search Tree (BST). For each node, all values in the left-subtree are \(<\) the node and all values in the right-subtree are \(>\) the node.

\[
\begin{align*}
50 & \\
30 & \quad 70 \\
\quad 9 & \quad 58 \\
\quad 32 & \quad 80
\end{align*}
\]

a. What is the order of node processing in a preorder traversal of the above BST?

b. What is the order of node processing in a postorder traversal of the above BST?

c. What is the order of node processing in an inorder traversal of the above BST?

d. Starting at the root, how would you find the node containing \(32\)?

e. Starting at the root, when would you discover that \(65\) is not in the BST?

f. Starting at the root, where would be the \textit{easiest} place to add \(65\)?

g. Where would we add \(5\) and \(33\)?