1. The textbook solves the coin-change problem with the following code (note the "set-builder-like" notation):

$\{c \mid c \in \text{coinValueList and } c \leq \text{change}\}$

```
def recMC(change, coinValueList):
    global backtrackingNodes
    backtrackingNodes += 1
    minCoins = change
    if change in coinValueList:
        return 1
    else:
        for i in [c for c in coinValueList if c <= change]:
            numCoins = 1 + recMC(change - i, coinValueList)
            if numCoins < minCoins:
                minCoins = numCoins
    return minCoins
```

Results of running this code:

```
Change Amount: 63 Coin types: [1, 5, 10, 25]
Run-time: 70.689 seconds
Fewest number of coins 6
Number of Backtracking Nodes: 67,716,925
```

I removed the fancy set-builder notation and replaced it with a simple `if`-statement check:

```
def recMC(change, coinValueList):
    global backtrackingNodes
    backtrackingNodes += 1
    minCoins = change
    if change in coinValueList:
       return 1
    else:
       for i in coinValueList:
           if i <= change:
               numCoins = 1 + recMC(change - i, coinValueList)
               if numCoins < minCoins:
                   minCoins = numCoins
    return minCoins
```

Results of running this code:

```
Change Amount: 63 Coin types: [1, 5, 10, 25]
Run-time: 45.815 seconds
Fewest number of coins 6
Number of Backtracking Nodes: 67,716,925
```

a) Why is the second version so much "faster"?

b) Why does it still take a long time?

2. To speed the recursive backtracking algorithm, we can prune unpromising branches. The general recursive backtracking algorithm for optimization problems (e.g., fewest number of coins) looks something like:

```
Backtrack( recursionTreeNode p ) {
   for each child c of p do                          # each c represents a possible choice
       if promising(c) then                          # c is "promising" if it could lead to a better solution
            if c is a solution that's better than best then   # check if this is the best solution found so far
                 best = c                            # remember the best solution
            else
                 Backtrack(c)                        # follow a branch down the tree
            end if
       end if
   end for
} // end Backtrack
```

General Notes about Backtracking:

- The depth-first nature of backtracking only stores information about the current branch being explored on the run-time stack, so the memory usage is "low" eventhough the # of recursion tree nodes might be exponential (2^n).
- Each node of the search-space (recursive-call) tree maintains the state of a partial solution. In general the partial solution state consists of potentially large arrays that change little between parent and child. To avoid having multiple copies of these arrays, a reference to a single "global" array can be maintained which is updated before we go down to the child (via a recursive call) and undone when we backtrack to the parent.

a) For the coin-change problem, what defines the current state of a search-space tree node?

b) When would a "child" tree node NOT be promising?

3. Consider the output of running the backtracking code with pruning (next page) twice with a change amount of 63 cents.

```
Change Amount: 63  Coin types: [1, 5, 10, 25]
Run-time: 0.036 seconds
Fewest number of coins 6
The number of each type of coins is:
number of 1-cent coins is 3
number of 5-cent coins is 0
number of 10-cent coins is 1
number of 25-cent coins is 2
Number of Backtracking Nodes: 4831
```

```
Change Amount: 63  Coin types: [25, 10, 5, 1]
Run-time: 0.003 seconds
Fewest number of coins 6
The number of each type of coins is:
number of 25-cent coins is 2
number of 10-cent coins is 1
number of 5-cent coins is 0
number of 1-cent coins is 3
Number of Backtracking Nodes: 310
```

a) Explain why ordering the coins from largest to smallest produced faster results.

b) For coins of [50, 25, 12, 10, 5, 1] typical timings:

Change Amount	Run-Time (seconds)	Number of Tree Nodes
399	8.88	2,015,539
409	55.17	12,093,221
419	318.56	72,558,646

Why the exponential growth in run-time?

4. As with Fibonacci, the coin-change problem can benefit from dynamic program since it was slow due to solving the same problems over-and-over again. Recall the general idea of dynamic programming:

- Solve smaller problems before larger ones
- store their answers
- look-up answers to smaller problems when solving larger subproblems, so each problem is solved only once

a) To solve the coin-change problem using dynamic programming, we need to answer the questions:

- What is the smallest problem?

- Where do we store the answers to the smaller problems?

```
backtrackingNodes = 0  # profiling variable to track number of state-space tree nodes

def solveCoinChange(changeAmt, coinTypes):

    def backtrack(changeAmt, numberOfEachCoinType, numberOfCoinsSoFar, solutionFound, bestFewestCoins, bestNumberOfEachCoinType):
        global backtrackingNodes
        backtrackingNodes += 1

        for index in range(len(coinTypes)):
            smallerChangeAmt = changeAmt - coinTypes[index]
            if promising(smallerChangeAmt, numberOfCoinsSoFar+1, solutionFound, bestFewestCoins):
                if smallerChangeAmt == 0:  # a solution is found
                    if (not solutionFound) or numberOfCoinsSoFar + 1 < bestFewestCoins: # check if its best
                        bestFewestCoins = numberOfCoinsSoFar+1
                        bestNumberOfEachCoinType = [] + numberOfEachCoinType
                        bestNumberOfEachCoinType[index] += 1
                        solutionFound = True
                else:
                    # call child with updated state information
                    smallerChangeAmtNumberOfEachCoinType = [] + numberOfEachCoinType
                    smallerChangeAmtNumberOfEachCoinType[index] += 1

                    solutionFound, bestFewestCoins, bestNumberOfEachCoinType = backtrack(smallerChangeAmt, smallerChangeAmtNumberOfEachCoinType,
                                                                                         numberOfCoinsSoFar + 1, solutionFound, bestFewestCoins,
                                                                                         bestNumberOfEachCoinType)
        return solutionFound, bestFewestCoins, bestNumberOfEachCoinType
    # end def backtrack

    def promising(changeAmt, numberOfCoinsReturned, solutionFound, bestFewestCoins):
        if changeAmt < 0:
            return False
        elif changeAmt == 0:
            return True
        else:  # changeAmt > 0
            if solutionFound and numberOfCoinsReturned+1 >= bestFewestCoins:
                return False
            else:
                return True

    # Body of solveCoinChange
    numberOfEachCoinType = []          # set-up initial "current state" information
    numberOfCoinsSoFar = 0
    solutionFound = False
    bestFewestCoins = -1
    bestNumberOfEachCoinType = None


    numberOfEachCoinType = []
    for coin in coinTypes:
        numberOfEachCoinType.append(0)
    numberOfCoinsSoFar = 0
    solutionFound = False
    bestFewestCoins = -1
    bestNumberOfEachCoinType = None


    solutionFound, bestFewestCoins, bestNumberOfEachCoinType = backtrack(changeAmt, numberOfEachCoinType, numberOfCoinsSoFar, solutionFound,
                                                                         bestFewestCoins, bestNumberOfEachCoinType)
    return bestFewestCoins, bestNumberOfEachCoinType
```

Dynamic Programming Coin-change Algorithm:

I. Fills an array fewestCoins from 0 to the amount of change. An element of fewestCoins stores the fewest number of coins necessary for the amount of change corresponding to its index value.

For 29-cents using the set of coin types {1, 5, 10, 12, 25, 50}, the dynamic programming algorithm would have previously calculated the fewestCoins for the change amounts of 0, 1, 2, ..., up to 28 cents.

II. If we record the best, first coin to return for each change amount (found in the "minimum" calculation) in an array bestFirstCoin, then we can easily recover the actual coin types to return.

fewestCoins[29] = minimum(fewestCoins[28], fewestCoins[24], fewestCoins[19], fewestCoins[17], fewestCoins[4]) + 1 = 2 + 1 = 3

a minimum for 29 given by 5-cent coin

fewestCoins: [4] = 4, [17] = 2, [19] = 4, [24] = 2, [28] = 4, [29] = 3 (reached from 29 by coins 25, 12, 10, 5, 1)

bestFirstCoin: [0] = 0, [12] = 12, [24] = 12, [29] = 5 (12-12=0, 24-12=12, 29-5=24)

Extract the coins in the solution for 29-cents from bestFirstCoin[29], bestFirstCoin[24], and bestFirstCoin[12]

b) Extend the lists through 32-cents.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
fewestCoins:	0	1	2	3	4	1	2	3	4	5	1	2	1	2	3	2	3	2	3	4	2	3	2	3	2	1	2	3	4	3			

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
bestFirstCoin:	0	1	1	1	1	5	1	1	1	1	10	1	12	1	1	5	1	5	1	1	10	1	10	1	12	25	1	1	1	5			

c) What coins are in the solution for 32-cents?

Data Structures (CS 1520) Lab 6 Name:________________

Objective: To gain experience with a simple, recursive, divide-and-conquer implementation and how to develop faster solutions using an iterative dynamic programming implementation

To start the lab: Download and unzip the file at: www.cs.uni.edu/~fienup/cs1520f18/labs/lab6.zip

Part A: We've seen several recursive "divide-and-conquer" algorithms: fibonacci and coin-change problem. The general idea of divide-and-conquer algorithms is:

- dividing the original problem it into small problem(s) (e.g., fib(n-1) and fib(n-2))
- solving the smaller problem(s) recursively
- combine the solution(s) to smaller problem(s) to solve the original problem (e.g., return fib(n-1) + fib(n-2))

Mathematics has several simple recursively defined functions. For example, the *factorial* function can be recursively defined as:

(1) $n! = n * (n - 1)!$ for $n \geq 1$, and
$0! = 1$

Implement a recursive `factorial(n)` function using this recursive definition and test it with several small examples (e.g., 3! = 3*2! = 3*2*1! = 3*2*1*0! = 3*2*1*1 = 6, and 5! = 5*4*3*2*1*1 = 120).

In Discrete Structures (CS 1800) you used (or will use) the binomial coefficient formula:

(2) $$C(n, k) = \frac{n!}{k!(n-k)!}$$

to calculate the number of combinations of "n choose k," i.e., the number of ways to choose k objects from n objects. For example, when calculating the number of unique 5-card hands from a standard 52-card deck (e.g., C(52, 5)) we need to calculate 52! / 5! 47! = 2,598,960.

Implement the function C(n, k) using formula (2) above and your recursive factorial function.

Completed your factorial and binomial coefficient functions, raise your hand and demonstrate your code.

Part B: One problem with using formula (2) in most languages is that n! grows very fast and overflows the integer representation before you can do the division to bring the value back to a value that can be represented. (NOTE: Python does not suffer from this problem, but lets pretend that it does.)
For example, when calculating C(52, 5) we need to calculate 52! / 5! 47!. However, the value of
52! = 80,658,175,170,943,878,571,660,636,856,403,766,975,289,505,440,883,277,824,000,000,000,000
is much, much bigger than can fit into a 64-bit integer representation. Fortunately, another way to view C(52, 5) is recursively by splitting the problem into two smaller problems by focusing on:

- the hands containing a specific card, say the ace of clubs, and
- the hands that do not contain the ace of clubs.

For those hands that do contain the ace of clubs, we need to choose 4 more cards from the remaining 51 cards, i.e., $C(51, 4)$. For those hands that do not contain the ace of clubs, we need to choose 5 cards from the remaining 51 cards, i.e., $C(51, 5)$. Therefore, $C(52, 5) = C(51, 4) + C(51, 5)$.

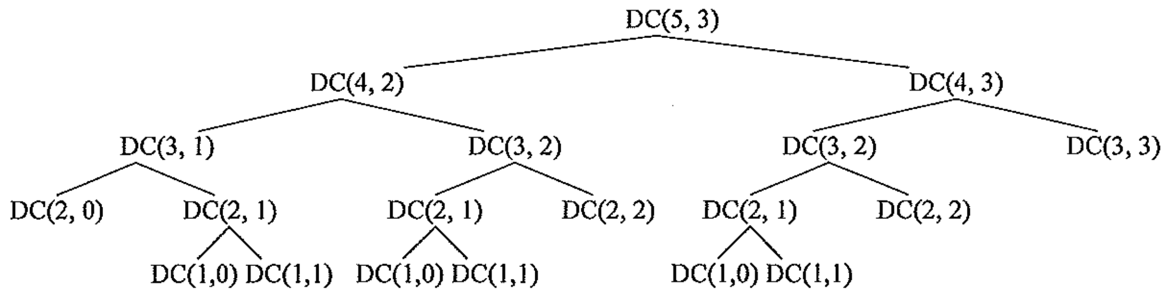
In general, **(NOTE: When implementing your recursive code, be sure to use DC for the recursive calls)**

(3) $C(n, k) = C(n-1, k-1) + C(n-1, k)$ for $1 \leq k \leq (n-1)$, and
$C(n, k) = 1$ for $k = 0$ or $k = n$

Implement the recursive "divide-and-conquer" binomial coefficient function using equation (3). Call your function `DC(n,k)` for "divide-and-conquer". Notice the difference in run-time between calculating the binomial coefficient using C(24, 12) vs. DC(24, 12), C(26, 13) vs. DC(26, 13), and C(28, 14) vs. DC(28, 14).

Completed your binomial coefficient function, DC, raise your hand and demonstrate your code.

Part C: Much of the slowness of your "divide-and-conquer" binomial coefficient function, DC(n, k), is due to redundant calculations performed due to the recursive calls. For example, the recursive calls associated with DC(5, 3) = 10 would be:



Pascal's triangle (named for the 17th-century French mathematician Blaise Pascal, and for whom the programming language Pascal was also named) is a "dynamic programming" approach to calculating binomial coefficients.

```
                     1                          Row #
                  1     1                         0
               1     2     1                      1
            1     3     3     1                   2
         1     4     6     4     1                3
      1     5     10    10    5     1             4
                     .                            5
                     .
```

Recall that dynamic programming solutions eliminate the redundancy of recursive divide-and-conquer algorithms by calculating the solutions to smaller problems first, storing their answers, and looking up their answers if later needed instead of recalculating it. Abstractly, Pascal's triangle relates to the binomial coefficient as in:

								Row #
C(0,0)								0
C(1,0)	C(1,1)							1
C(2,0)	C(2,1)	C(2,2)						2
C(3,0)	C(3,1)	C(3,2)	C(3,3)					3
C(4,0)	C(4,1)	C(4,2)	C(4,3)	C(4,4)				4
C(5,0)	C(5,1)	C(5,2)	C(5,3)	C(5,4)	C(5,5)			5
⋮								⋮
			C(n-1,k-1)	C(n-1,k)				n-1
			↘ + ↓					
C(n,0)	C(n,1)	C(n,2)	...	C(n,k)		C(n, n-1)	C(n,n)	n

For Part C, your job is to implement the "dynamic programming" binomial coefficient function using a Python list of lists to store Pascal's triangle and loops (no recursion needed). Call your function `DP(n,k)` for "dynamic programming". **Hints for Part C:**

- Review the dynamic programming fibonacci example from Lecture 9. File `lab6/fibonacci.py` contains the recursive divide-and-conquer, and two dynamic programming versions of fibonacci. The first dynamic programming version, `fib_DP`, stores the answers to all of the smaller problems. The second dynamic programming version, `fib_DP2`, reduces the amount of memory used by only storing the answers for the previous two smaller problems.

Notice the difference in run-time between calculating the binomial coefficient using DC(24, 12) vs. DP(24, 12), DC(26, 13) vs. DP(26, 13), and DC(28, 14) vs. DP(28, 14).

Completed your binomial coefficient function, DP, raise your hand and demonstrate your code.

Part D: EXTRA CREDIT

- Your function `DP_Extra_Credit(n,k)` should use the idea of `fib_DP2` to avoid storing all the answers to the smaller problems. **Notice that the calculation of the next row in the picture above only needs the previous row and none of the older rows.**

Data Structures Lecture 14 Name:_______________

1. Consider the following sequential search (linear search) code:

<table>
<tr><th>Textbook's Listing 5.1</th><th>Faster sequential search code</th></tr>
<tr><td>

```
def sequentialSearch(alist, item):
    """ Sequential search of unorder list """
    pos = 0
    found = False

    while pos < len(alist) and not found:
        if alist[pos] == item:
            found = True
        else:
            pos = pos+1

    return found
```

</td><td>

```
def linearSearch(aList, target):
    """Returns the index of target in aList
       or -1 if target is not in aList"""
    for position in range(len(aList)):
        if target == aList[position]:
            return position
    return -1
```

</td></tr>
</table>

a) What is the *basic operation* of a search?

b) For the following `aList` value, which `target` value causes `linearSearch` to loop the fewest ("best case") number of times?

	0	1	2	3	4	5	6	7	8	9	10
aList:	10	15	28	42	60	69	75	88	90	93	97

c) For the above `aList` value, which `target` value causes `linearSearch` to loop the most ("worst case") number of times?

d) For a *successful search* (i.e., `target` value in `aList`), what is the "average" number of loops?

<table>
<tr><th>Textbook's Listing 5.2</th><th>Faster sequential search code</th></tr>
<tr><td>

```
def orderedSequentialSearch(alist, item):
    """ Sequential search of order list """
    pos = 0
    found = False
    stop = False
    while pos < len(alist) and not found and not stop:
        if alist[pos] == item:
            found = True
        else:
            if alist[pos] > item:
                stop = True
            else:
                pos = pos+1

    return found
```

</td><td>

```
def linearSearchOfSortedList(target, aList):
    """Returns the index position of target in
    sorted aList or -1 if target is not in aList"""
    breakOut = False
    for position in range(len(aList)):
        if target <= aList[position]:
            breakOut = True
            break

    if not breakOut:
        return -1
    elif target == aList[position]:
        return position
    else:
        return -1
```

</td></tr>
</table>

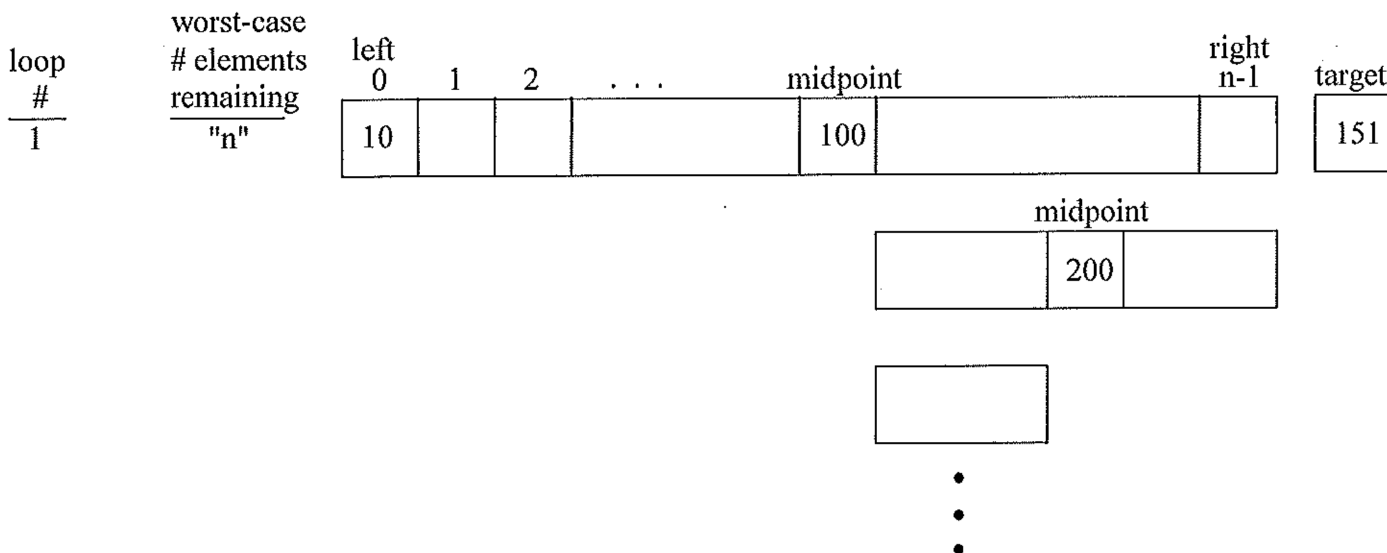
e) The above version of linear search assumes that `aList` is sorted in ascending order. When would this version perform better than the original `linearSearch` at the top of the page?

2. Consider the following binary search code:

Textbook's Listing 5.3	Faster binary search code

<pre>def binarySearch(alist, item):
    first = 0
    last = len(alist)-1
    found = False
 	
    while first<=last and not found:
        midpoint = (first + last)//2
        if alist[midpoint] == item:
            found = True
        else:
            if item < alist[midpoint]:
                last = midpoint-1
            else:
                first = midpoint+1
 	
    return found</pre>
	<pre>def binarySearch(target, lyst):
	    """Returns the position of the target
	       item if found, or -1 otherwise."""
	    left = 0
	    right = len(lyst) - 1
	    while left <= right:
	        midpoint = (left + right) // 2
	        if target == lyst[midpoint]:
	            return midpoint
	        elif target < lyst[midpoint]:
	            right = midpoint - 1
	        else:
	            left = midpoint + 1
	    return -1</pre>

a) "Trace" binary search to determine the worst-case basic total number of comparisons?



b) What is the worst-case big-oh for binary search?

c) What is the best-case big-oh for binary search?

d) What is the average-case (expected) big-oh for binary search?

e) If the list size is 1,000,000, then what is the maximun number of comparisons of list items on a *successful search*?

f) If the list size is 1,000,000, then how many comparisons would you expect on an *unsuccessful search*?

3. Hashing Motivation and Terminology:

a) Sequential search of an array or linked list follows the same search pattern for any given target value being searched for, i.e., scans the array from one end to the other, or until the target is found.
If n is the number of items being searched, what is the average and worst case big-oh notation for a sequential search?

average case $O($ $)$

worst case $O($ $)$

b) Similarly, binary search of a sorted array (or AVL tree) always uses a fixed search strategy for any given target value. For example, binary search always compares the target value with the middle element of the remaining portion of the array needing to be searched.
If n is the number of items being searched, what is the average and worst case big-oh notation for a search?

average case $O($ $)$

worst case $O($ $)$

Hashing tries to achieve average constant time (i.e., $O(1)$) searching by using the target's value to calculate where in the array/Python list (called the *hash table*) it should be located, i.e., each target value gets its own search pattern. The translation of the target value to an array index (called the target's *home address*) is the job of the *hash function*. A *perfect hash function* would take your set of target values and map each to a unique array index.

Set of Keys	Hash function
John Doe	hash(John Doe) = 6
Philip East	hash(Philip East) = 3
Mark Fienup	hash(Mark Fienup) = 5
Ben Schafer	hash(Ben Schafer) = 8

Hash Table Array

Index	Name	Phone
0		
1		
2		
3	Philip East	3-2939
4		
5	Mark Fienup	3-5918
6	John Doe	3-4567
7		
8	Ben Schafer	3-2187
9		
10		

a) If n is the number of items being searched and we had a perfect hash function, what is the average and worst case big-oh notation for a search?

average case $O($ $)$

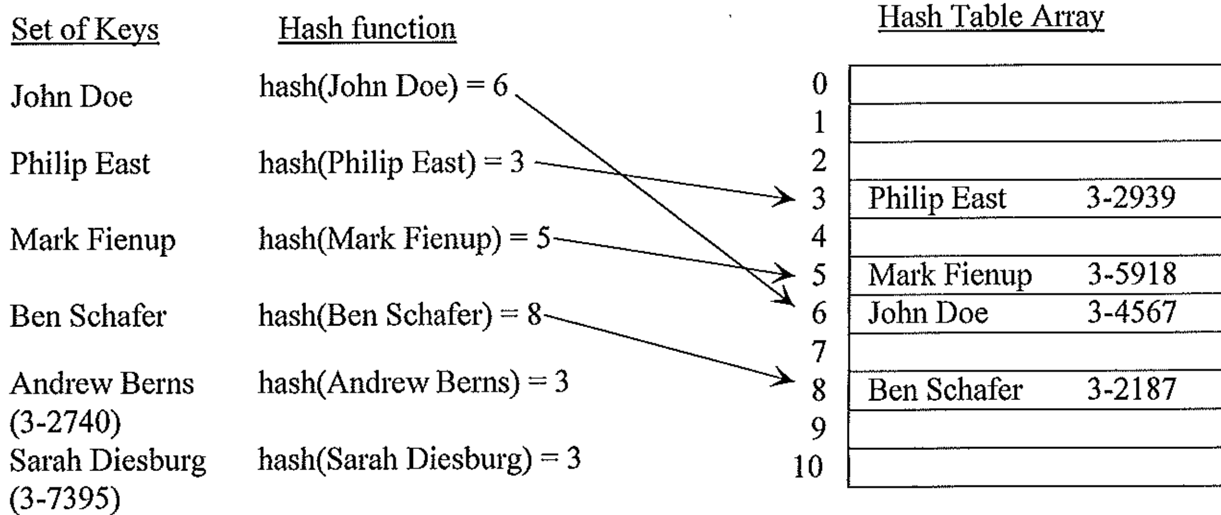
worst case $O($ $)$

4. Unfortunately, perfect hash functions are a rarity, so in general many target values might get mapped to the same hash-table index, called a *collision*.

Collisions are handled by two approaches:

- *open-address* with some *rehashing* strategy: Each hash table home address holds at most one target value. The first target value hashed to a specify home address is stored there. Later targets getting hashed to that home address get rehashed to a different hash table address. A simple rehashing strategy is *linear probing* where the hash table is scanned circularly from the home address until an empty hash table address is found.
- *chaining, closed-address*, or *external chaining*: all target values hashed to the same home address are stored in a data structure (called a *bucket*) at that index (typically a linked list, but a BST or AVL-tree could also be used). Thus, the hash table is an array of linked list (or whatever data structure is being used for the buckets)

5. Consider the following examples using *open-address* approach with a simple rehashing strategy of *linear probing* where the hash table is scanned circularly from the home address until an empty hash table address is found.

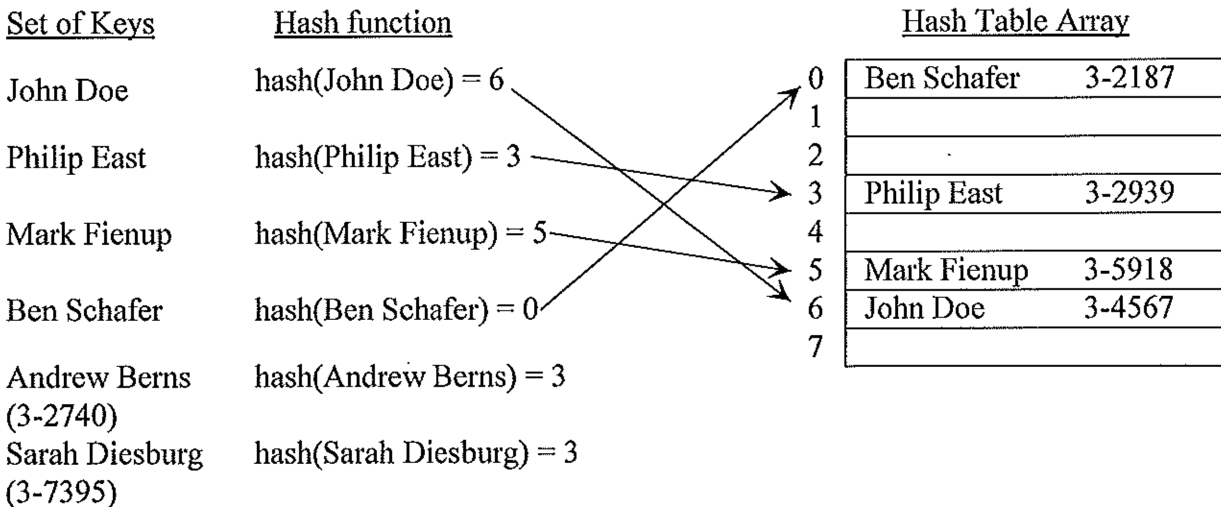


a) Assuming open-address with linear probing where would Andrew Berns and then Sarah Diesburg be placed?

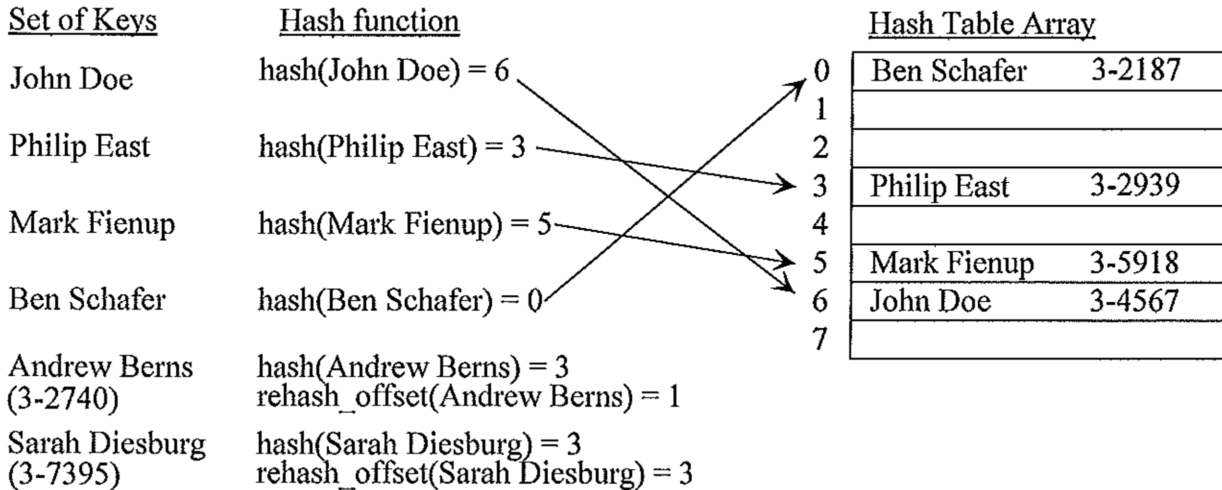
Common rehashing strategies include the following.

Rehash Strategy	Description
linear probing	Check next spot (counting circularly) for the first available slot, i.e., (home address + (rehash attempt #)) % (hash table size)
quadratic probing	Check the square of the attempt-number away for an available slot, i.e., (home address + ((rehash attempt #)^2 +(rehash attempt #))//2) % (hash table size), where the hash table size is a power of 2
double hashing	Use the target key to determine an offset amount to be used each attempt, i.e., (home address + (rehash attempt #) * offset) % (hash table size), where the hash table size is a power of 2 and the offset hash returns an odd value between 1 and the hash table size

b) Assume quadratic probing, insert "Andrew Berns" and "Sarah Diesburg" into the hash table.



c) Assume double hashing, insert "Andrew Berns" and "Sarah Diesburg" into the hash table.



d) For the above double-hashing example, what would be the sequence of hashing and rehashing addresses tried for Sarah Diesburg if the table was full? For the above example, (home address + (rehash attempt #) * offset) % (hash table size) would be: (3 + (rehash attempt #) * 3) % 8

Rehash Attempt #	0	1	2	3	4	5	6	7	8	9	10
Address											

e) Indicate whether each of the following rehashing strategies suffer from primary or secondary clustering.

- *primary clustering* - keys mapped to a home address follow the same rehash pattern
- *secondary clustering* - rehash patterns from initially different home addresses merge together

Rehash Strategy	**Description**	**Suffers from:**	
		primary clustering	**secondary clustering**
linear probing	Check next spot (counting circularly) for the first available slot, i.e., (home address + (rehash attempt #)) % (hash table size)		
quadratic probing	Check a square of the attempt-number away for an available slot, i.e., (home address + ((rehash attempt #)^2 +(rehash attempt #))/2) % (hash table size), where the hash table size is a power of 2		
double hashing	Use the target key to determine an offset amount to be used each attempt, i.e., (home address + (rehash attempt #) * offset) % (hash table size), where the hash table size is a power of 2 and the offset hash returns an odd value between 1 and the hash table size		

6. Let λ be the *load factor* (# item/hash table size). The average probes with **linear probing** for insertion or unsuccessful search is: $\left(\frac{1}{2}\right)\left(1+\left(\frac{1}{(1-\lambda)^2}\right)\right)$. The average for successful search is: $\left(\frac{1}{2}\right)\left(1+\left(\frac{1}{(1-\lambda)}\right)\right)$.

a) Why is an unsuccessful search worse than a successful search?

The average probes with **quadratic probing** for insertion or unsuccessful search is: $\left(\frac{1}{1-\lambda}\right) - \lambda - \log_e(1-\lambda)$

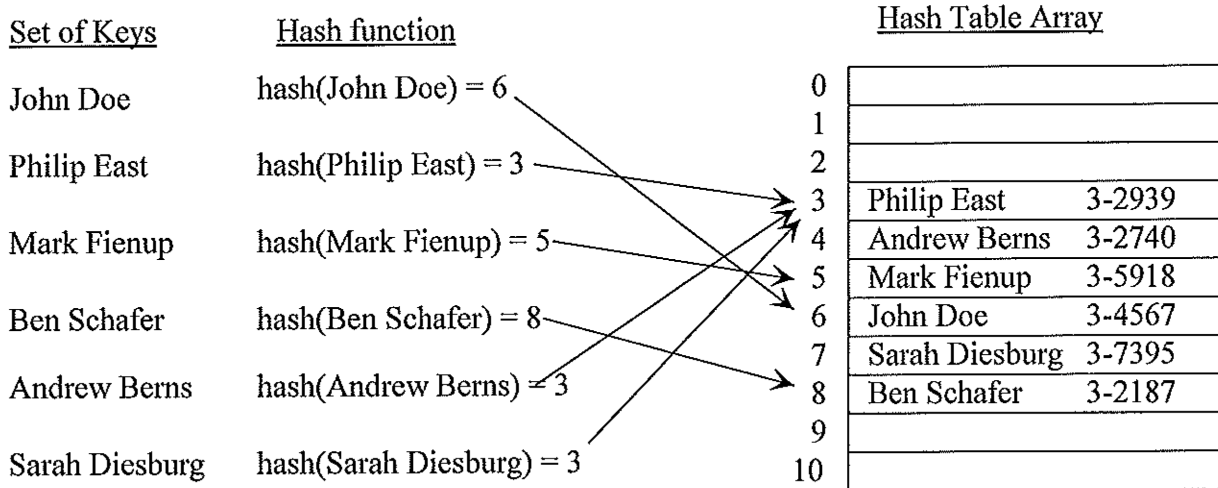
The average probes with quadratic probing for successful search is: $1 - \left(\frac{\lambda}{2}\right) - \log_e(1-\lambda)$

Consider the following table containing the average number probes for various load factors:

Probing Type	**Search outcome**	**Load Factor, λ**				
		0.25	**0.5**	**0.67**	**0.8**	**0.99**
Linear Probing	unsuccessful	1.39	2.50	5.09	13.00	5000.50
	successful	1.17	1.50	2.02	3.00	50.50
Quadratic Probing	unsuccessful	1.37	2.19	3.47	5.81	103.62
	successful	1.16	1.44	1.77	2.21	5.11

b) Why do you suppose the "general rule of thumb" in hashing tries to keep the load factor between 0.5 and 0.67?

7. Allowing deletions from an open-address hash table complicates the implementation. Assuming linear probing we might have the following

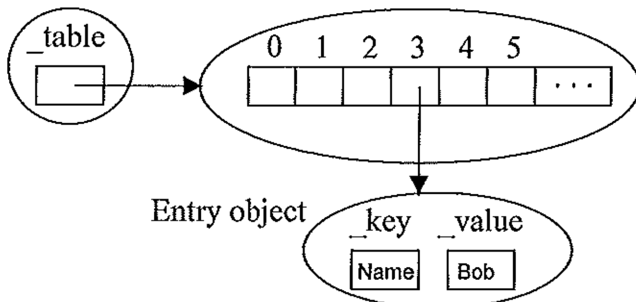


a) If "Mark Fienup" is deleted, how will we find Sarah Diesburg?

b) How might we fix this problem?

1. The Map/Dictionary abstract data type (ADT) stores key-value pairs. The key is used to look up the data value.

Method call	Class Name	Description
`d = ListDict()`	`__init__(self)`	Constructs an empty dictionary
`d["Name"] = "Bob"`	`__setitem__(self,key,value)`	Inserts a key-value entry if `key` does not exist or replaces the old value with `value` if `key` exists.
`temp = d["Name"]`	`__getitem__(self,key)`	Given a `key` return it value or `None` if `key` is not in the dictionary
`del d["Name"]`	`__delitem__(self,key)`	Removes the entry associated with `key`
`if "Name" in d:`	`__contains__(self,key)`	Return `True` if `key` is in the dictionary; return `False` otherwise
`for k in d:`	`__iter__(self)`	Iterates over the keys in the dictionary
`len(d)`	`__len__(self)`	Returns the number of items in the dictionary
`str(d)`	`__str__(self)`	Returns a string representation of the dictionary




```
from entry import Entry

class ListDict(object):
    """Dictionary implemented with a Python list."""

    def __init__(self):
        self._table = []

    def __getitem__(self, key):
        """Returns the value associated with key or
        returns None if key does not exist."""
        entry = Entry(key, None)
        try:    # NOTE: Python list index method
                # errors on unsuccessful search
            index = self._table.index(entry)
            return self._table[index].getValue()
        except:
            return None

    def __delitem__(self, key):
        """Removes the entry associated with key."""
        entry = Entry(key, None)
        try:    # NOTE: Python list index method
                # errors on unsuccessful search
            index = self._table.index(entry)
            self._table.pop(index)
        except:
            return

    def __str__(self):
        """Returns string repr. of the dictionary"""
        resultStr = "{"
        for item in self._table:
            resultStr = resultStr + " " + str(item)
        return resultStr + " }"

    def __iter__(self):
        """Iterates over keys of the dictionary"""
        for item in self._table:
            yield item.getKey()
        raise StopIteration
```

```
class Entry(object):
    """A key/value pair."""

    def __init__(self, key, value):
        self._key = key
        self._value = value

    def getKey(self):
        return self._key

    def getValue(self):
        return self._value

    def setValue(self, newValue):
        self._value = newValue

    def __eq__(self, other):
        if not isinstance(other, Entry):
            return False
        return self._key == other._key

    def __str__(self):
        return str(self._key) + ":" + str(self._value)
```

a) Complete the code for the `__contains__` method.

```
def __contains__(self, key):
```

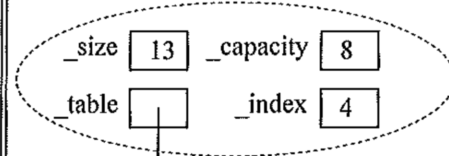
b) Complete the code for the `__setitem__` method.

```
def __setitem__(self, key, value):
```

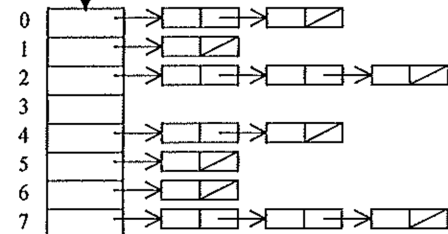
2. Dictionary implementation using hashing with chaining -- an UnorderedList object at each slot in the hash table.

```
from entry import Entry
from unordered_linked_list import UnorderedList

class ChainingDict(object):
    """Dictionary implemented using hashing with chaining."""

    def __init__(self, capacity = 8):
        self._capacity = capacity
        self._table = []
        for index in range(self._capacity):
            self._table.append(UnorderedList())
        self._size = 0
        self._index = None

    def __contains__(self, key):
        """Returns True if key is in the dictionary or
        False otherwise."""
        self._index = abs(hash(key)) % self._capacity
        entry = Entry(key, None)

        return self._table[self._index].search(entry)

    def __getitem__(self, key):
        """Returns the value associated with key or
        returns None if key does not exist."""
        if key in self:
            entry = Entry(key, None)
            entry = self._table[self._index].remove(entry)
            self._table[self._index].add(entry)
            return entry.getValue()
        else:
            return None

    def __delitem__(self, key):
        """Removes the entry associated with key."""
        if key in self:
            entry = Entry(key, None)
            entry = self._table[self._index].remove(entry)
            self._size -= 1

    def __setitem__(self, key, value):
        """Inserts an entry with key/value if key
        does not exist or replaces the existing value
        with value if key exists."""
        entry = Entry(key, value)
        if key in self:
            entry = self._table[self._index].remove(entry)
            entry.setValue(value)
        else:
            self._size += 1
        self._table[self._index].add(entry)

    def __len__(self):
        return self._size

    def __str__(self):
        result = "HashDict: capacity = " + \
                 str(self._capacity) + ", load factor = " + \
                 str(len(self) / self._capacity)
        for i in range(self._capacity):
            result += "\nRow " + str(i)+": "+str(self._table[i])
        return result

    def __iter__(self):
        """Iterates over the keys of the dictionary"""
```

ChainingDict Object



Python list of UnorderedList objects containing Entrys



a) In `__getitem__`, why is the `entry = Entry(key, None)` object created?

b) In `__getitem__`, where does `self._index` receive its value?

c) What single modification was needed to the `UnorderedList`'s `remove` method?

d) Complete the `__iter__` method.

Data Structures (CS 1520) Lab 7 - Hashing Name:________________

Objective: To experiment with searching and get a feel for the performance of hashing.

To start the lab: Download and unzip the file lab7.zip

Part A:

a) Open and run the timeLinearSearch.py program that times the LinearSearch algorithm imported from LinearSearch.py. Obverse that it creates a list, evenList, that holds 10,000 sorted, even values (e.g., evenList = [0, 2, 4, 6, 8, ..., 19996, 19998]). It then times the searching for target values from 0, 1, 2, 3, 4, ..., 19998, 19999 so half of the searches are successful and half are unsuccessful. How long does it take to linear search for target values from 0, 1, 2, 3, 4, ..., 19998, 19999?

b) Open and run the timeBinarySearch.py program that times the binarySearch algorithm imported from binarySearch.py. How long does it take to binary search for target values from 0, 1, 2, 3, 4, ..., 19998, 19999?

c) Open and run the timeListDictSearch.py program that times the ListDict dictionary ADT in list_dictionary.py. The ListDict implementation uses a single Python list for storing dictionary entries. The timeListDictSearch.py program adds the 10,000 even values (i.e., 0, 2, 4, 6, 8, ..., 19996, 19998) to a ListDict object, and then times the searching for target values from 0, 1, 2, 3, 4, ..., 19998, 19999 so half of the searches are successful and half are unsuccessful. How long does it take to search for target values from 0, 1, 2, 3, 4, ..., 19998, 19999 in the ListDict?

d) Open and run the timeChainingDictSearch.py program that times the ChainingDict dictionary ADT in chaining_dictionary.py. The timeChainingDictSearch.py program adds the 10,000 even values (i.e., 0, 2, 4, 6, 8, ..., 19996, 19998) to an ChainingDict with 16,384 slots (i.e., load factor of 0.61), and then times the searching for target values from 0, 1, 2, 3, 4, ..., 19998, 19999 so half of the searches are successful and half are unsuccessful. How long does it take to search for target values from 0, 1, 2, 3, 4, ..., 19998, 19999 in the ChainingDict?

e) Explain the relative performance results of searching using linear search, binary search, a ListDict, and ChainingDict. (Think about their big-oh notations and their constants of proportionalities "c")

f) The Python `for` loop allows traversal of built-in data structures (strings, lists, tuple, etc) by an *iterator*. To accomplish this with ***our*** data structures we need to include an `__iter__` method (e.g., ListDict class from Lecture 15 at http://www.cs.uni.edu/~fienup/cs1520f18/lectures/lec15_questions.pdf). In general an `__iter__` method, must loop down the data structure and `yield` each item in the data structure. When done, the `__iter__` needs to `raise StopIteration`. See the end of `UnorderedList` and `ListDict` classes for examples of their `__iter__` methods. Complete the `__iter__` code for the `ChainingDict` and `OpenAddrHashDict`classes.

After you have completed the above timings, questions and code, raise your hand and explain your answers.

Part B:

a) Open and run the timeOpenAddrHashDictSearch.py program that times the OpenAddrHashDict dictionary ADT in open_addr_hash_dictionary.py. The timeOpenAddrHashDictSearch.py program adds the 10,000 even values (i.e., 0, 2, 4, 6, 8, ..., 19996, 19998) to an OpenAddrHashDict with 16,384 (2^{14}) slots (i.e., load factor of 0.61) using linear probing, and then times the searching for target values from 0, 1, 2, 3, 4, ..., 19998, 19999 so half of the searches are successful and half are unsuccessful. How long does it take to search for target values from 0, 1, 2, 3, 4, ..., 19998, 19999 in the OpenAddrHashDict?

b) **Place the even values (i.e., 0, 2, 4, 6, 8, ..., 19996, 19998) in the hash table below.** Value 0 is stored at home address 0, value 2 is stored at home address 2, ..., value 16,382 is stored at home address 16,382, but values 16,384 to 19,998 will have collisions. Now, think about the number of probes needed to searching for target values from 0, 1, 2, 3, 4, ..., 19998, 19999. Why does the above timing of searching for target values from 0, 1, 2, 3, 4, ..., 19998, 19999 take so long with a load factor of only 0.61?

	self._table
0	
1	
2	
3	
4	
5	
6	
7	
16,380	
16,381	
16,382	
16,383	

c) Experiment with changing the load factor of the HashTable by increasing the hash table size to 32,768 (2**15) for a load factor of 0.31, and 65,536 for a load factor of 0.15. Completing the following table:

Linear Probing	Hash Table Size (Load Factor)		
	16,384 (0.61)	32,768 (0.31)	65,536 (0.15)
Execution time with 10,000 items in hash table (seconds)			

d) In timeOpenAddrHashDictSearch.py modify the construction of evenHashTable so it uses quadratic probing instead of linear probing (i.e., evenHashTable = OpenAddrHashTable(2**14, hash, **False**)). Completing the following table:

Quadratic Probing	Hash Table Size (Load Factor)		
	16,384 (0.61)	32,768 (0.31)	65,536 (0.15)
Execution time with 10,000 items in hash table (seconds)			

e) Explain why quadratic probing performs better than linear probing.

After you have performed the timings and answered the questions, raise your hand and explain your answers. Remember to save your lab7 files for later usage on homework assignments!

1. The Dictionary implementation using open-address hashing was the `OpenAddrHashDict` class in `lab7.zip`.

```
from entry import Entry

class OpenAddrHashDict(object):
    EMPTY = None  # class variables shared by all objects of the class
    DELETED = True

    def __init__(self, capacity = 8, hashFunction = hash,
                 linear = True):
        self._table = [OpenAddrHashDict.EMPTY] * capacity
        self._size = 0
        self._hash = hashFunction
        self._homeIndex = -1
        self._actualIndex = -1
        self._linear = linear
        self._probeCount = 0

    def __getitem__(self, key):
        """Returns the value associated with key or
        returns None if key does not exist."""
        if key in self:
            return self._table[self._actualIndex].getValue()
        else:
            return None

    def __delitem__(self, key):
        """Removes the entry associated with key."""
        if key in self:
            self._table[self._actualIndex] = OpenAddrHashDict.DELETED
            self._size -= 1

    def __setitem__(self, key, value):
        """Inserts an entry with key/value if key does not exist or
           replaces the existing value with value if key exists."""
        entry = Entry(key, value)
        if key in self:
            self._table[self._actualIndex] = entry
        else:
            self._table[self._actualIndex] = entry
            self._size += 1

    def __contains__(self, key):
        """Return True if key is in the dictionary; return False otherwise"""
        entry = Entry(key, None)
        self._probeCount = 0
        # Get the home index
        self._homeIndex = abs(self._hash(key)) % len(self._table)
        rehashAttempt = 0
        index = self._homeIndex

        # Stop searching when an empty cell is encountered
        while rehashAttempt < len(self._table):
            self._probeCount += 1
            if self._table[index] == OpenAddrHashDict.EMPTY:
                self._actualIndex = index
                return False # An empty cell is found, so key not found
            elif self._table[index] == entry:
                self._actualIndex = index
                return True

            # Calculate the index and wrap around to first position if necessary
            rehashAttempt += 1
            if self._linear:
                index = (self._homeIndex + rehashAttempt) % len(self._table)
            else:  # Quadratic probing
                index = (self._homeIndex + (rehashAttempt ** 2+ rehashAttempt) // 2) % len(self._table)

        return False   # tried all the slots in the hash table and did not find key

    def __len__(self):
        return self._size

    def __str__(self):
        resultStr = "{"
        for item in self._table:
            if not item in (OpenAddrHashDict.EMPTY,OpenAddrHashDict.DELETED):
                resultStr = resultStr + " " + str(item)
        return resultStr + " }"

    def __iter__(self):
        """Iterates over the keys of the dictionary"""
```

a) Complete the `__iter__` method.

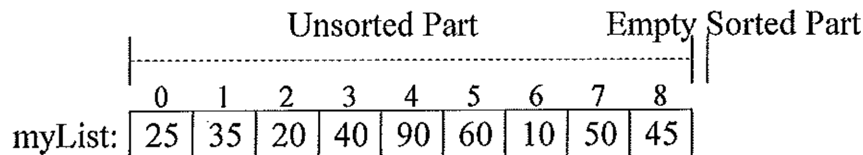
2. All *simple sorts* consist of two nested loops where:

- the **outer loop** keeps track of the dividing line between the sorted and unsorted part with the sorted part growing by one in size each iteration of the outer loop.
    - the **inner loop's** job is to do the work to extend the sorted part's size by one.

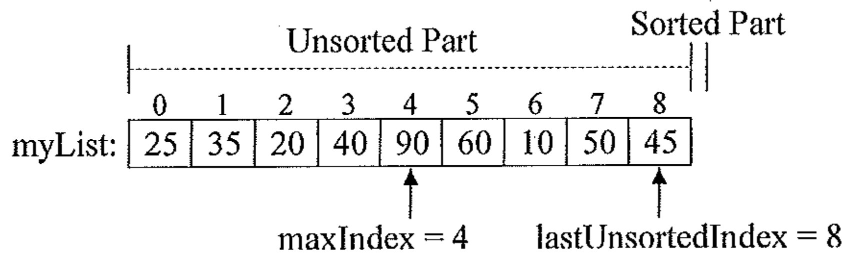
Initially, the sorted part is typically empty. The simple sorts differ in how their inner loops perform their job.

Selection sort is an example of a simple sort. Selection sort's inner loop scans the unsorted part of the list to find the maximum item. The maximum item in the unsorted part is then exchanged with the last unsorted item to extend the sorted part by one item.

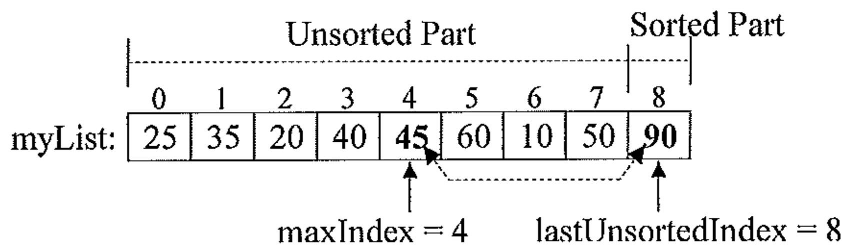
At the start of the first iteration of the outer loop, initial list is completely unsorted:



The inner loop scans the unsorted part and determines that the index of the maximum item, maxIndex = 4.



After the inner loop (but still inside the outer loop), the item at maxIndex is exchanged with the item at lastUnsortedIndex. Thus, extending the Sorted Part of the list by one item.



a) Write the code for the outer loop

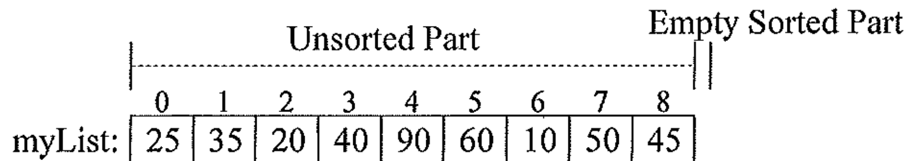
b) Write the code for the inner loop to scan the unsorted part of the list to determine the index of the maximum item

c) Write the code to exchange the list items at positions maxIndex and lastUnsortedIndex.

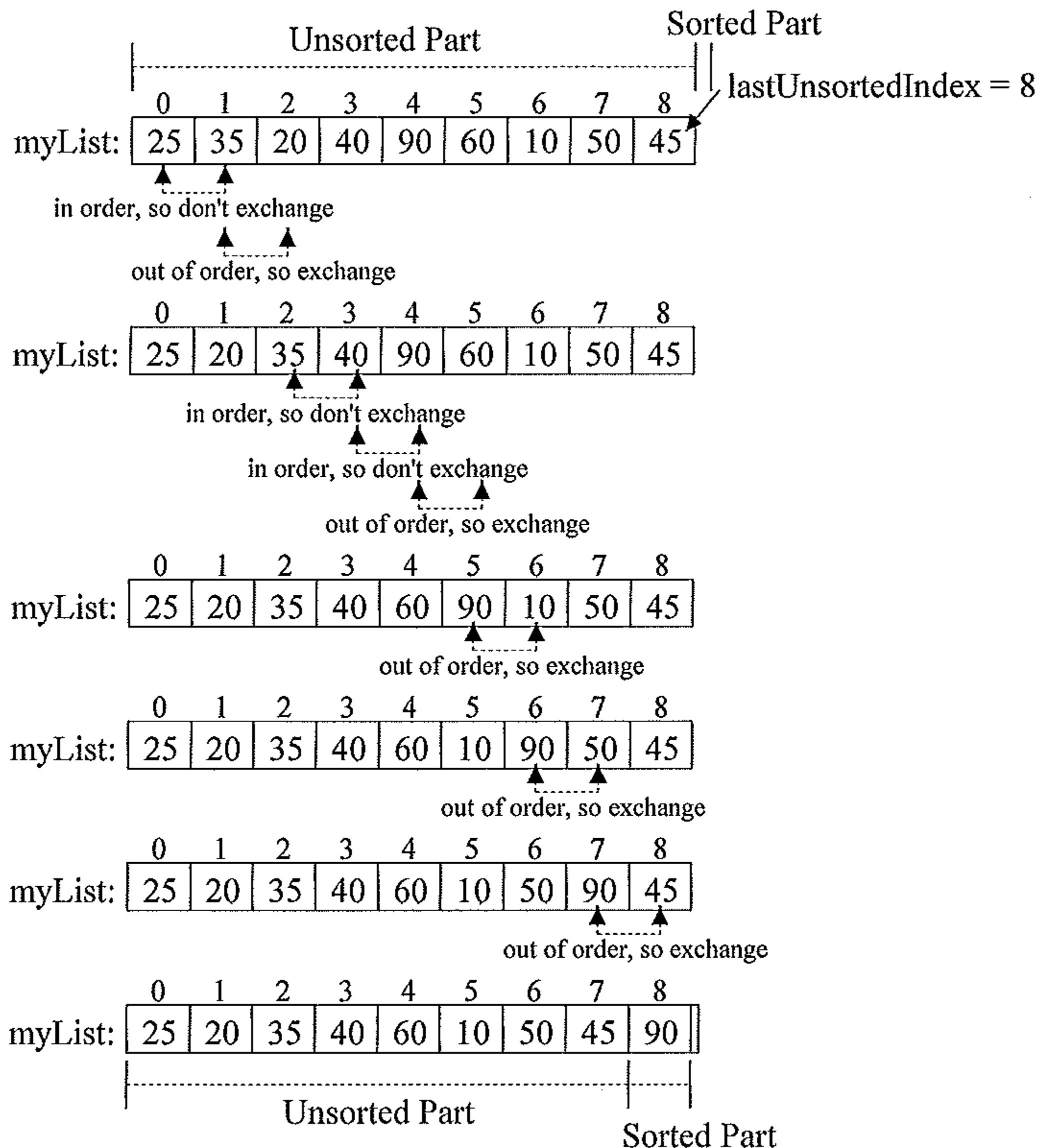
d) What is the big-oh notation for selection sort?

3. *Bubble sort* is another example of a simple sort. Bubble sort's inner loop scans the unsorted part of the list comparing adjacent items. If it finds adjacent items out of order, then it exchanges them. This causes the largest item to "bubble" up to the "top" of the unsorted part of the list.

At the start of the first iteration of the outer loop, initial list is completely unsorted:



The inner loop scans the unsorted part by comparing adjacent items and exchanging them if out of order.



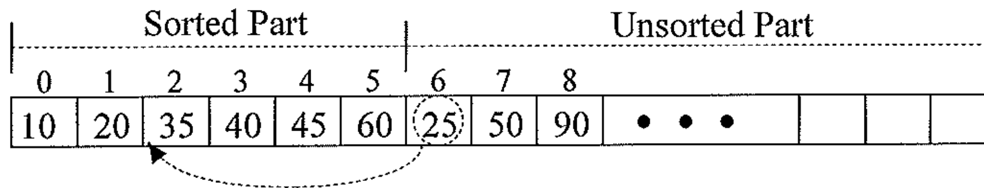
After the inner loop (but still inside the outer loop), there is nothing to do since the exchanges occurred inside the inner loop.

a) What would be the worst-case big-oh of bubble sort?

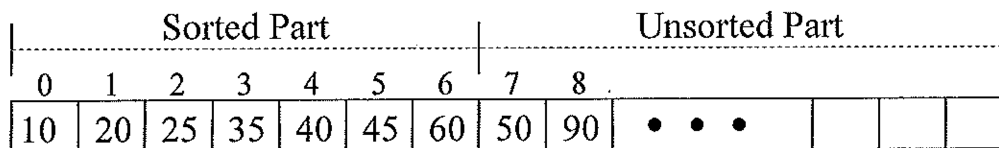
b) What would be true if we scanned the unsorted part and didn't need to do any exchanges?

4. Another simple sort is called insertion sort. Recall that in a simple sort:
- the outer loop keeps track of the dividing line between the sorted and unsorted part with the sorted part growing by one in size each iteration of the outer loop.
  - the inner loop's job is to do the work to extend the sorted part's size by one.

After several iterations of insertion sort's outer loop, a list might look like:



In insertion sort the inner-loop takes the "first unsorted item" (25 at index 6 in the above example) and "inserts" it into the sorted part of the list "at the correct spot." After 25 is inserted into the sorted part, the list would look like:



Code for insertion is given below:

```
def insertionSort(myList):
    """Rearranges the items in myList so they are in ascending order"""

    for firstUnsortedIndex in range(1,len(myList)):
        itemToInsert = myList[firstUnsortedIndex]

        testIndex = firstUnsortedIndex - 1

        while testIndex >= 0 and myList[testIndex] > itemToInsert:
            myList[testIndex+1] = myList[testIndex]
            testIndex = testIndex - 1

        # Insert the itemToInsert at the correct spot
        myList[testIndex + 1] = itemToInsert
```

a) What is the purpose of the `testIndex >= 0` while-loop comparison?

b) What initial arrangement of items causes the is the overall worst-case performance of insertion sort?

c) What is the worst-case $O(\,)$ notation for the number of item moves?

d) What is the worst-case $O(\,)$ notation for the number of item comparisons?

e) What initial arrangement of items causes the is the overall best-case performance of insertion sort?

f) What is the best-case $O(\,)$ notation for insertion sort?

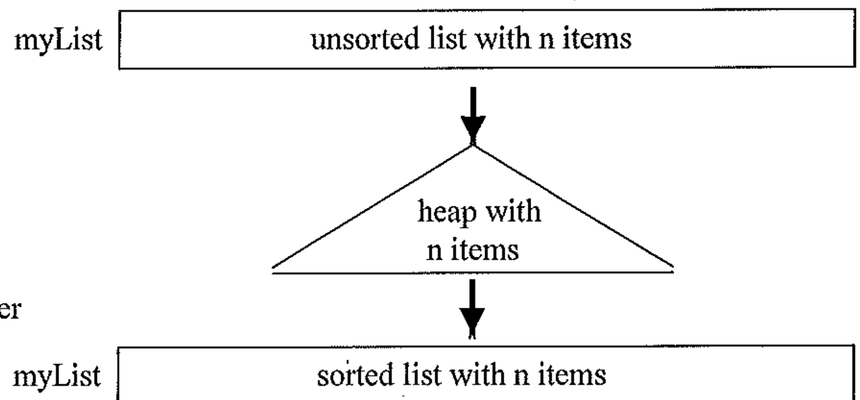
Data Structures Lecture 17 Name:________________

1. So far, we have looked at simple sorts consisting of nested loops. The # of inner loop iterations $n*(n-1)/2$ is $O(n^2)$. Consider using a min-heap to sort a list. (methods: `BinHeap()`, `insert(item)`, `delMin()`, `isEmpty()`, `size()`)

a) If we insert all of the list elements into a min-heap, what would we easily be able to determine?

General idea of Heap sort:

1. Create an empty heap
2. Insert all n list items into heap
3. delMin heap items back to list in sorted order



b) What is the overall $O(\,)$ for heap sort?

2. Another way to do better than the simple sorts is to employ divide-and-conquer (e.g., Merge sort and Quick Sort). Recall the idea of **Divide-and-Conquer** algorithms. Solve a problem by:

- dividing problem into smaller problem(s) of the same kind
- solving the smaller problem(s) recursively
- use the solution(s) to the smaller problem(s) to solve the original problem

In general, a problem can be solved recursively if it can be broken down into smaller problems that are identical in structure to the original problem.

a) What determines the "size" of a sorting problem?

b) How might we break the original problem down into smaller problems that are identical?

c) What base case(s) (i.e., trival, non-recursive case(s)) might we encounter with recursive sorts?

d) How do you combine the answers to the smaller problems to solve the original sorting problem?

e) Consider why a recursive sort might be more efficient. Assume that I had a simple n^2 sorting algorithm with $n = 100$, then there is roughly $100^2 / 2$ or 5,000 amount of work. Suppose I split the problem down into two smaller sorting problems of size 50.

- If I run the n^2 algorithm on both smaller problems of size 50, then what would be the approximate amount of work?

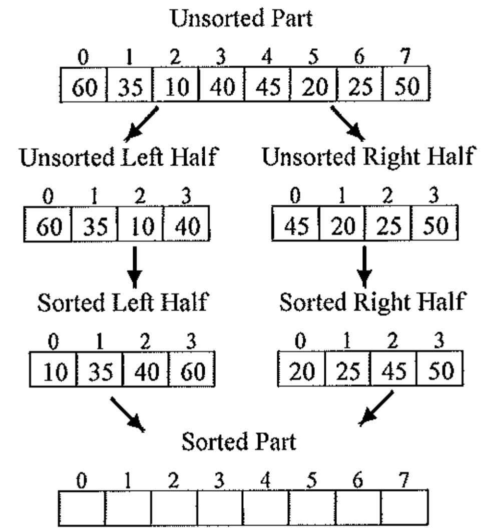
- If I further solve the problems of size 50 by splitting each of them into two problems of size 25, then what would be the approximate amount of work?

3. The general idea merge sort is as follows. Assume "n" items to sort.

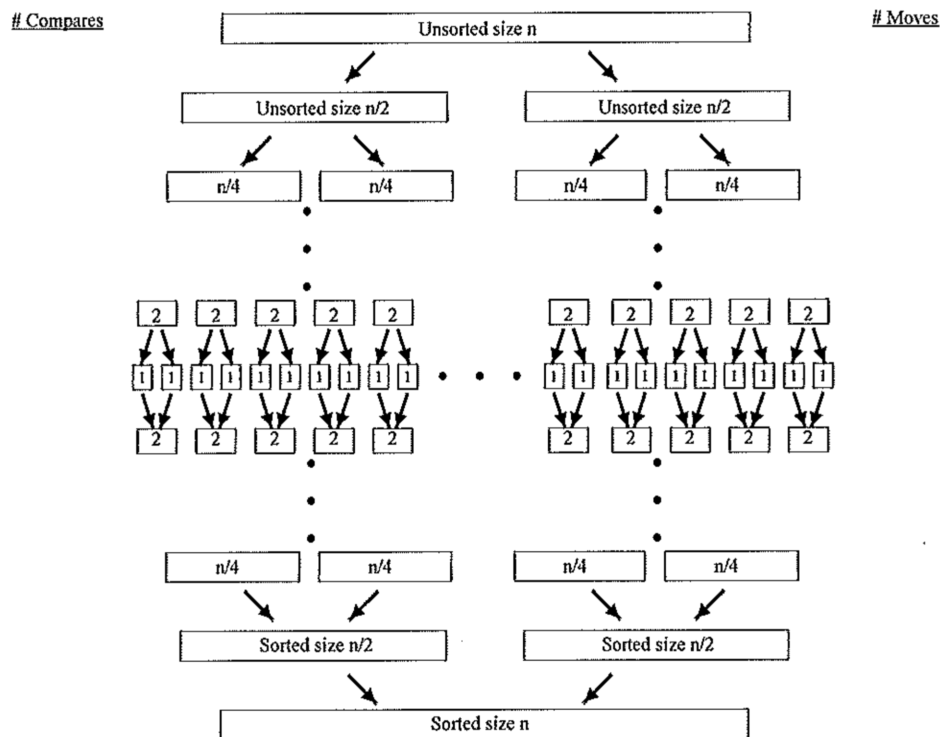
- Split the unsorted part in half to get two smaller sorting problems of about equal size = n/2
- Solve both smaller problems recursively using merge sort
- "Merge" the solutions to the smaller problems together to solve the original sorting problem of size n

a) Fill in the merged Sorted Part in the diagram.

b) Describe how you filled in the sorted part in the above example?



4. Merge sort is substantially faster than the simple sorts. Let's analyze the number of comparisons and moves of merge sort. Assume "n" items to sort.



a) On each level of the above diagram write the WORST-CASE number of comparisons and moves for that level.

b) What is the WORST-CASE total number of comparisons and moves for the whole algorithm (i.e., add all levels)?

c) What is the big-oh for worst-case?

5. *Quick sort* general idea is as follows.

- Select a "random" item in the unsorted part as the *pivot*
- Rearrange (*partitioning*) the unsorted items such that:
- Quick sort the unsorted part to the left of the pivot
- Quick sort the unsorted part to the right of the pivot

All items < to Pivot	Pivot Item (Pivot Index)	All items >= to Pivot

a) Given the following `partition` function which returns the index of the pivot after this rearrangement, complete the recursive `quicksortHelper` function.

```
def partition(lyst, left, right):
    # Find the pivot and exchange it with the last item
    middle = (left + right) // 2
    pivot = lyst[middle]
    lyst[middle] = lyst[right]
    lyst[right] = pivot
    # Set boundary point to first position
    boundary = left
    # Move items less than pivot to the left
    for index in range(left, right):
        if lyst[index] < pivot:
            temp = lyst[index]
            lyst[index] = lyst[boundary]
            lyst[boundary] = temp
            boundary += 1
    # Exchange the pivot item and the boundary item
    temp = lyst[boundary]
    lyst[boundary] = lyst[right]
    lyst[right] = temp
    return boundary
```

```
def quicksort(lyst):

    quicksortHelper(lyst, 0, len(lyst) - 1)

def quicksortHelper(lyst, left, right):
```

b) For the list below, trace the first call to partition and determine the resulting list, and value returned.

	0	1	2	3	4	5	6	7	8	left	right	index	boundary	pivot
lyst:	54	26	93	17	50	31	44	55	20	0	8			

b) What initial arrangement of the list would cause partition to perform the most amount of work?

c) Let "n" be the number of items between left and right. What is the worst-case *O*( ) for partition?

d) What would be the overall, worst-case $O(\ )$ for Quick Sort?

e) Ideally, the pivot item splits the list into two equal size problems. What would be the big-oh for Quick Sort in the best case?

f) What would be the big-oh for Quick Sort in the average case?

g) The textbook's `partition` code (Listing 5.15 on page 225) selects the first item in the list as the pivot item. However, the above `partition` code selects the middle item of the list to be the pivot. What advantage does selecting the middle item as the pivot have over selecting the first item as the pivot?

Data Structures (CS 1520) Lab 8 Name:________________________

Objectives: You will gain experience:

- get a feel for simple sorts: selection, bubble, and insertion sorts
- get a feel for advanced sorts: heap, quick, and merge sorts

To start the lab: Download and unzip the file: http://www.cs.uni.edu/~fienup/cs1520f18/labs/lab8.zip
The lab8.zip file you downloaded and extracted contains the following sorting algorithms which all sort in ascending order (i.e., from smallest to largest):

- bubbleSort.py - bubble sort code which **does not** check if it can stop early
- bubbleSortB.py - bubble sort code which stops early if no swapping is needed during a scan of the unsorted part
- insertionSort.py - the insertion sort
- selectionSort.py - the selection sort code we developed in class

Each program runs the sorting algorithm several time with different initial orderings of 10,000 list items. The initial orderings of items are: descending order, ascending order, random order, and random order again to check for consistence. Complete the following timings by running the each program.

Timings of Sorting Algorithms on 10,000 items (seconds)				
Type of sorting algorithm	Initial Ordering of Items			
	Descending	Ascending	Random order 1	Random order 2
bubbleSort.py				
bubbleSortB.py				
insertionSort.py				
selectionSort.py				

Study the code and answer the following questions about the sorting algorithms:

a) Why does the bubbleSort algorithm take less time on the ascending ordered list than the descending ordered list?

b) Why does the bubbleSortB algorithm take A LOT less time on the ascending ordered list than the descending ordered list?

c) Why does the insertionSort algorithm take A LOT less time on the ascending ordered list than the descending ordered list?

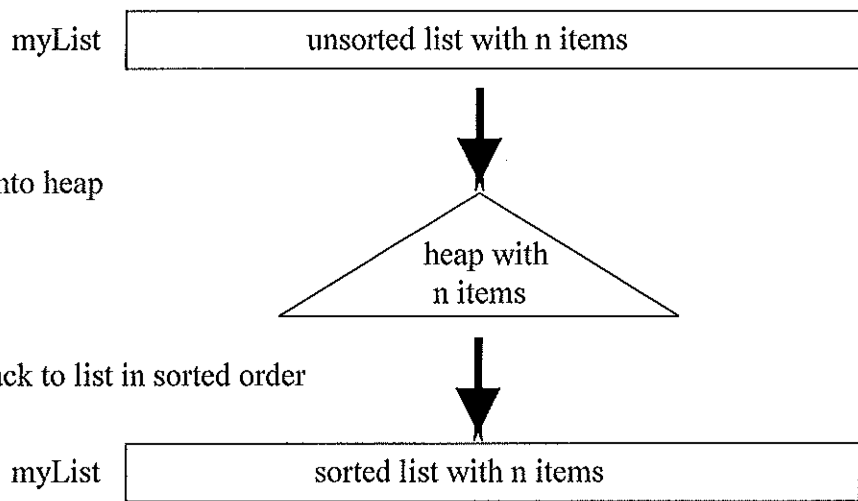
d) Why does the insertionSort algorithm take less time on the descending ordered list than the bubbleSort algorithm on the descending ordered list?

e) Why does the selectionSort algorithm take less time on the descending ordered list than the insertionSort algorithm on the descending ordered list?

After you have answered the above questions, raise your hand and explain your answers.

Part B:

a) Complete the heap sort function in `lab8/heapSort.py` which contains the template for the heap sort algorithm discussed in class. Recall the steps of the algorithm:

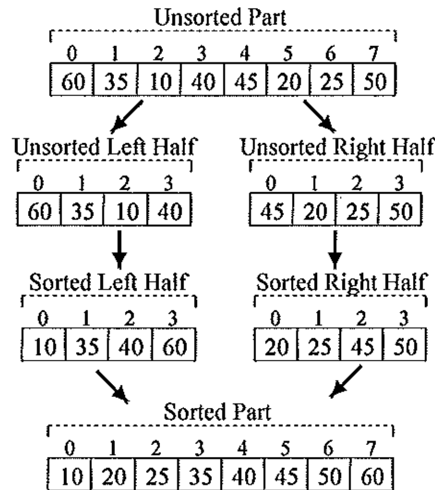


b) Time the heap sorting algorithm using `lab8/timeHeapSort.py` on 100,000 random items, 200,000 random items, and 400,000 random items.

# Items	Your Heap Sort Timing
100,000	
200,000	
400,000	

c) Explain the $O(\,)$ for your heap sort algorithm?

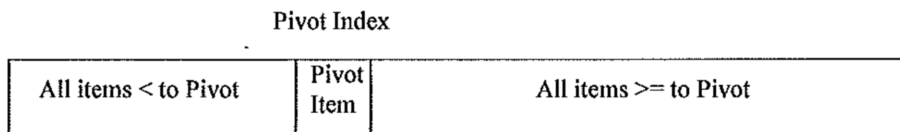
d) The general idea merge sort is as follows. Assume "n" items to sort.
- Split the unsorted part in half to get two smaller sorting problems of about n/2
- Solve both smaller problem recursively using merge sort
- "Merge" the solution to the smaller problems together to solve the original sorting problem of size n



The textbook's merge sort is in `mergesort.py`. Use the `timeMergeSort.py` program to run merge sort on a list of random items. Complete the following timing table:

Random # Items	Textbook's Merge Sort Timings
100,000	
200,000	
400,000	

e) Recall the general idea of *Quick sort* is as follows. Assume "n" items to sort.
- Select a "random" item in the unsorted part as the *pivot*
- Rearrange (called *partitioning*) the unsorted items such that:



- Quick sort the unsorted part to the left of the pivot
- Quick sort the unsorted part to the right of the pivot

The lecture 17 quick sort is in `quicksort.py`. Use the `timeQuickSort.py` program to run quick sort on a list of random items. Complete the following timings to get a feel for the "speed" of quicksort.

# Items	Lecture 17 Quick Sort Timings
100,000	
200,000	
400,000	

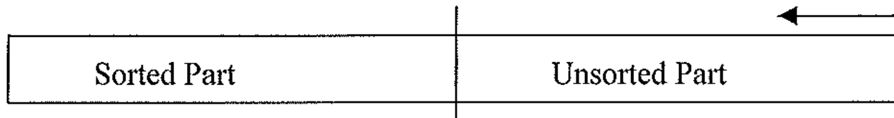
All three advanced sorting algorithms are $O(n\log_2 n)$ on initially random data. Why do you suppose quick sort is the fastest advanced sort on random items?

After you have completed the above times and answered the above question, raise your hand and explain your answers.

Part C: EXTRA CREDIT

a) Write (pencil-and-paper below) a variation of bubble sort that:

- sorts in descending order (largest to smallest)
- builds the sorted part on the left-hand side of the list, i.e.,



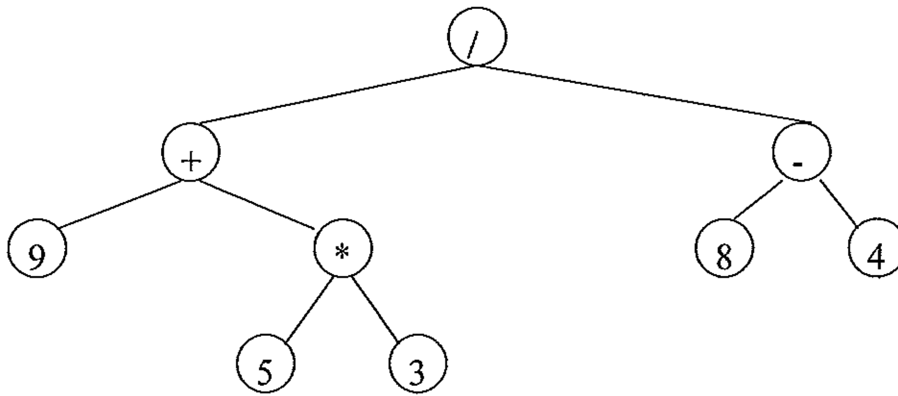
Inner loop scans from right to left across the unsorted part swapping adjacent items that are "out of order"

(Your code does NOT need to stop early if a scan of the unsorted part has no swaps)

```
def bubbleSortC(myList):
```

b) Implement and test your `bubbleSortC` code.

1. Consider the parse tree for (9 + (5 * 3)) / (8 - 4):



a) Indentify the following items in the above tree:

- *node* containing "*"
- *edge* from node containing "-" to node containing "8"
- *root* node
- *children* of the node containing "+"
- *parent* of the node containing "3"
- *siblings* of the node containing "*"
- *leaf* nodes of the tree
- *subtree* who's root is node contains "+"
- *path* from node containing "+" to node containing "5"
- *branch* from root node to "3"

b) Mark the *levels* of the tree (level is the number of edges on the path from the root)

c) What is the *height* (max. level) of the tree?

2. In Python an easy way to implement a tree is as a list of lists where a tree look like:

["node value", remaining items are subtrees for the node each implemented as a list of lists]

Complete the list-of-lists representation look like for the above parse tree:

['/', ['+',], ['-',]]

3. Consider a "linked" representations of a BinaryTree. For the expression ((4 + 5) * 7), the binary tree would be:

```
class BinaryTree:
    def __init__(self,rootObj):
        self.key = rootObj
        self.leftChild = None
        self.rightChild = None
```

key '*' leftChild rightChild

key '+' leftChild rightChild

key 7 leftChild rightChild

key 4 leftChild rightChild

key 5 leftChild rightChild

```
import operator
class BinaryTree:
    def __init__(self,rootObj):
        self.key = rootObj
        self.leftChild = None
        self.rightChild = None

    def insertLeft(self,newNode):
        if self.leftChild == None:
            self.leftChild=BinaryTree(newNode)
        else:
            t = BinaryTree(newNode)
            t.left = self.leftChild
            self.leftChild = t

    def insertRight(self,newNode):
        if self.rightChild == None:
            self.rightChild=BinaryTree(newNode)
        else:
            t = BinaryTree(newNode)
            t.right = self.rightChild
            self.rightChild = t

    def isLeaf(self):
        return ((not self.leftChild) and
                (not self.rightChild))

    def getRightChild(self):
        return self.rightChild

    def getLeftChild(self):
        return self.leftChild

    def setRootVal(self,obj):
        self.key = obj

    def getRootVal(self,):
        return self.key

    def inorder(self):
        if self.leftChild:
            self.leftChild.inorder()
        print(self.key)
        if self.rightChild:
            self.rightChild.inorder()

    def postorder(self):
        if self.leftChild:
            self.leftChild.postorder()
        if self.rightChild:
            self.rightChild.postorder()
        print(self.key)
```

a) Fix the `insertLeft` and `insertRight` code:
(Listing 6.6 and 6.7 are wrong in the text on pp. 242-3)

```
def preorder(self):
    print(self.key)
    if self.leftChild:
        self.leftChild.preorder()
    if self.rightChild:
        self.rightChild.preorder()

def printexp(self):
    if self.leftChild:
        print('(', end=' ')
        self.leftChild.printexp()
    print(self.key, end=' ')
    if self.rightChild:
        self.rightChild.printexp()
        print(')', end=' ')

def postordereval(self):
    opers = {'+':operator.add, '-':operator.sub,
             '*':operator.mul, '/':operator.truediv}
    res1 = None
    res2 = None
    if self.leftChild:
        res1 = self.leftChild.postordereval()
    if self.rightChild:
        res2 = self.rightChild.postordereval()
    if res1 and res2:
        return opers[self.key](res1,res2)
    else:
        return self.key
```

Some corresponding external (non-class) functions:

```
def inorder(tree):
    if tree != None:
        inorder(tree.getLeftChild())
        print(tree.getRootVal())
        inorder(tree.getRightChild())

def printexp(tree):
    if tree.leftChild:
        print('(', end=' ')
        printexp(tree.getLeftChild())
    print(tree.getRootVal(), end=' ')
    if tree.rightChild:
        printexp(tree.getRightChild())
        print(')', end=' ')

def height(tree):
    if tree == None:
        return -1
    else:
        return 1 +
          max(height(tree.leftChild),
              height(tree.rightChild))
```

```
def printexp(tree):
    sVal = ""
    if tree:
        sVal = '(' + printexp(tree.getLeftChild())
        sVal = sVal + str(tree.getRootVal())
        sVal = sVal + printexp(tree.getRightChild()) + ')'
    return sVal

def postordereval(tree):
    opers = {'+':operator.add, '-':operator.sub,
             '*':operator.mul, '/':operator.truediv}
    res1 = None
    res2 = None
    if tree:
        res1 = postordereval(tree.getLeftChild())
       res2 = postordereval(tree.getRightChild())
       if res1 and res2:
           return opers[tree.getRootVal()](res1,res2)
       else:
           return tree.getRootVal()
```

b) If `myTree` is the `BinaryTree` object for the expression: ((4 + 5) * 7), what gets printed by a calls to:

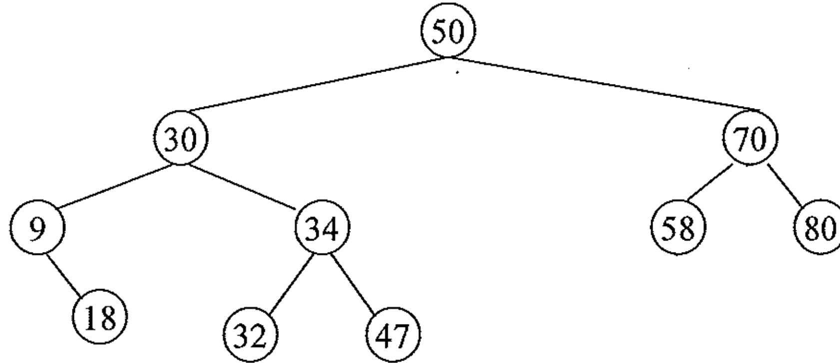
`myTree.inorder()`	`myTree.preorder()`	`myTree.postorder()`	`inorder(myTree)`

c) If `myTree` is the `BinaryTree` object for the expression: ((4 + 5) * 7), what gets printed by a call to `myTree.printexp()`?

d) If `myTree` is the `BinaryTree` object for the expression: ((4 + 5) * 7), what gets returned by a call to `myTree.postordereval()`?

e) Write the `height` method for the `BinaryTree` class.

4. Consider the Binary Search Tree (BST). For each node, all values in the left-subtree are < the node and all values in the right-subtree are > the node.



a. What is the order of node processing in a preorder traveral of the above BST?

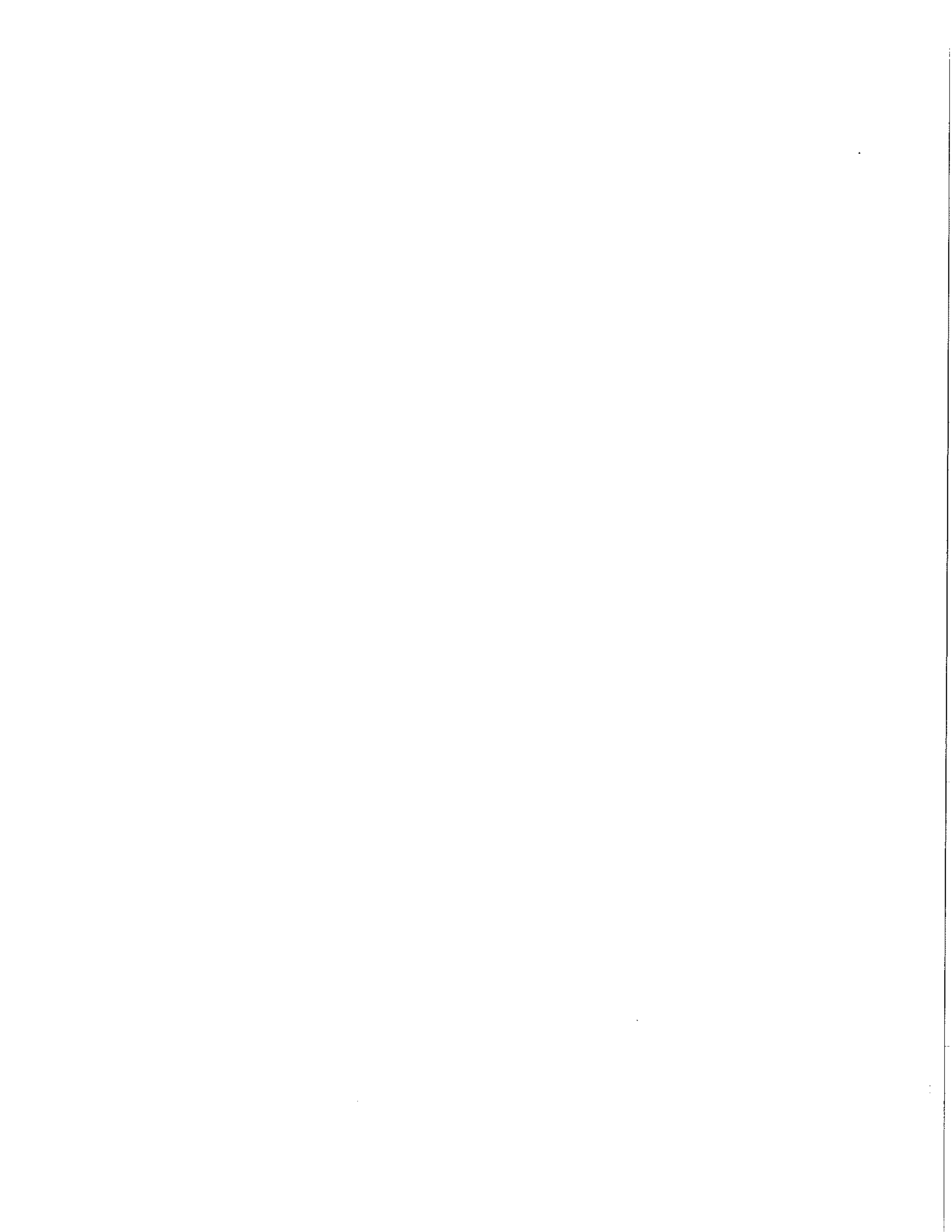
b. What is the order of node processing in a postorder traveral of the above BST?

c. What is the order of node processing in a inorder traveral of the above BST?
d. Starting at the root, how would you find the node containing "32"?

e. Starting at the root, when would you discover that "65" is not in the BST?

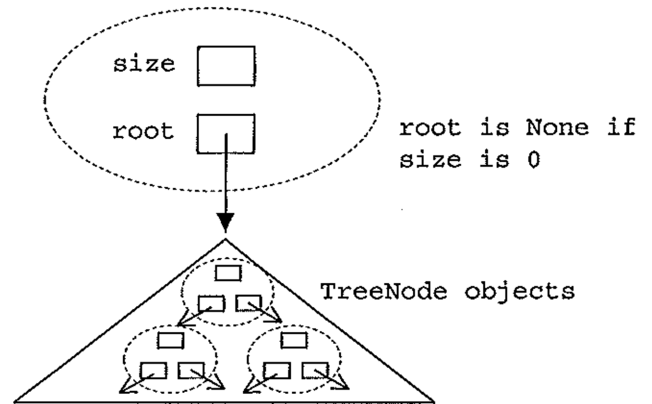
f. Starting at the root, where would be the "easiest" place to add "65"?

g. Where would we add "5" and "33"?



1. Consider the partial TreeNode class and partial BinarySearchTree class.

```
class TreeNode:
   def __init__(self,key,val,left=None,right=None,
                                          parent=None):
        self.key = key
        self.payload = val
        self.leftChild = left
        self.rightChild = right
        self.parent = parent

    def hasLeftChild(self):
        return self.leftChild

    def hasRightChild(self):
        return self.rightChild

    def isLeftChild(self):
        return self.parent and \
               self.parent.leftChild == self

    def isRightChild(self):
        return self.parent and \
               self.parent.leftChild == self

    def isRoot(self):
        return not self.parent

    def isLeaf(self):
      return not (self.rightChild or self.leftChild)

    def hasAnyChildren(self):
        return self.rightChild or self.leftChild

    def hasBothChildren(self):
        return self.rightChild and self.leftChild

    def replaceNodeData(self,key,value,lc,rc):
        self.key = key
        self.payload = value
        self.leftChild = lc
        self.rightChild = rc
        if self.hasLeftChild():
            self.leftChild.parent = self
        if self.hasRightChild():
            self.rightChild.parent = self

    def __iter__(self):
       if self:
          if self.hasLeftChild():
               for elem in self.leftChild:
                  yield elem
          yield self.key
          if self.hasRightChild():
               for elem in self.rightChild:
                  yield elem
```

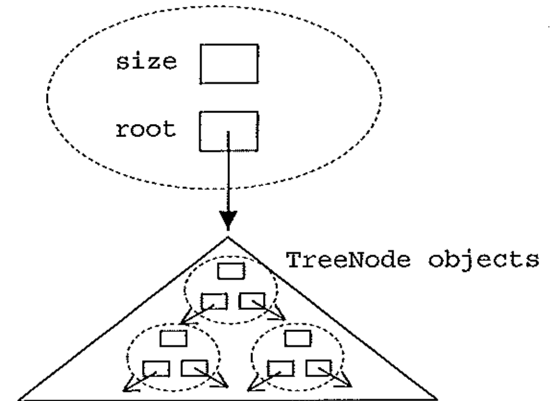



```
class BinarySearchTree:
  def __init__(self):
      self.root = None
      self.size = 0

  def length(self):
      return self.size

  def __len__(self):
      return self.size

  def __iter__(self):
      return self.root.__iter__()

  def __str__(self):
   """Returns a string representation of the tree
      rotated 90 degrees counter-clockwise"""

     def strHelper(root, level):
        resultStr = ""
        if root:
           resultStr += strHelper(root.rightChild,
                                  level+1)
           resultStr += "|  " * level
           resultStr += str(root.key) + "\n"
           resultStr += strHelper(root.leftChild,
                                  level+1)
        return resultStr

    return strHelper(self.root, 0)
```

a) How do the BinarySearchTree __iter__ and __str__ methods work?

More partial TreeNode class and partial BinarySearchTree class.

```
class BinarySearchTree:
    . . .
    def __contains__(self,key):
        if self._get(key,self.root):
            return True
        else:
            return False

    def get(self,key):
        if self.root:
            res = self._get(key,self.root)
            if res:
                return res.payload
            else:
                return None
        else:
            return None

    def _get(self,key,currentNode):
        if not currentNode:
            return None
        elif currentNode.key == key:
            return currentNode
        elif key < currentNode.key:
            return self._get(key,currentNode.leftChild)
        else:
            return self._get(key,currentNode.rightChild)

    def __getitem__(self,key):
        return self.get(key)

    def __setitem__(self,k,v):
        self.put(k,v)

    def put(self,key,val):
        if self.root:
            self._put(key,val,self.root)
        else:
            self.root = TreeNode(key,val)
        self.size = self.size + 1

    def _put(self,key,val,currentNode):
        if key < currentNode.key:
            if currentNode.hasLeftChild():


            else:



        elif




        else:
```



b) The `_get` method is the "work horse" of BST search. It recursively walks `currentNode` down the tree until it finds `key` or becomes `None`

In English, what are the base and recursive cases?

c) What is the `put` method doing?

d) Complete the recursive `_put` method.

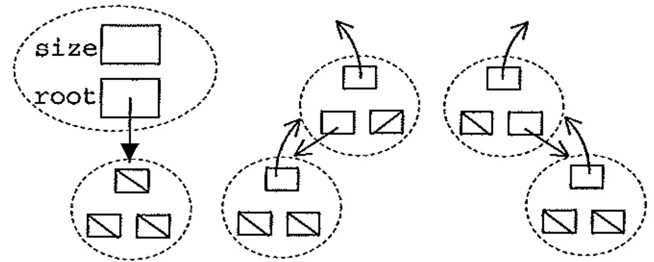
e) Draw the "shape" of the BST after `put`s of: 50, 60, 30, 70, 90, 40, 65

f) If "n" items are in the BST, what is `put`'s: Best-case $O($ $)$? Worst-case $O($ $)$? Average-case $O($ $)$?

2. More partial TreeNode class and partial BinarySearchTree class.

```
class BinarySearchTree:
  . . .
 def delete(self,key):
   if self.size > 1:
       nodeToRemove = self._get(key,self.root)
       if nodeToRemove:
           self.remove(nodeToRemove)
           self.size = self.size-1
       else:
           raise KeyError('Error, key not in tree')
   elif self.size == 1 and self.root.key == key:
       self.root = None
       self.size = self.size - 1
   else:
       raise KeyError('Error, key not in tree')

 def __delitem__(self,key):
     self.delete(key)


 def remove(self,currentNode):
   if currentNode.isLeaf(): #leaf
     if currentNode == currentNode.parent.leftChild:
         currentNode.parent.leftChild = None
     else:
         currentNode.parent.rightChild = None
   elif currentNode.hasBothChildren(): #interior
     succ = currentNode.findSuccessor()
     succ.spliceOut()
     currentNode.key = succ.key
     currentNode.payload = succ.payload

   else: # this node has one child
     if currentNode.hasLeftChild():
       if currentNode.isLeftChild():
           currentNode.leftChild.parent = currentNode.parent
           currentNode.parent.leftChild = currentNode.leftChild
       elif currentNode.isRightChild():
           currentNode.leftChild.parent = currentNode.parent
           currentNode.parent.rightChild = currentNode.leftChild
       else:
           currentNode.replaceNodeData(currentNode.leftChild.key,
                                  currentNode.leftChild.payload,
                                  currentNode.leftChild.leftChild,
                                  currentNode.leftChild.rightChild)

     else:
       if currentNode.isLeftChild():
           currentNode.rightChild.parent = currentNode.parent
           currentNode.parent.leftChild = currentNode.rightChild
       elif currentNode.isRightChild():
           currentNode.rightChild.parent = currentNode.parent
           currentNode.parent.rightChild = currentNode.rightChild
       else:
           currentNode.replaceNodeData(currentNode.rightChild.key,
                                  currentNode.rightChild.payload,
                                  currentNode.rightChild.leftChild,
                                  currentNode.rightChild.rightChild)
```

a) Update picture where we delete a leaf.



b) Where in the code is each handled?

c) Draw all pictures deleting all nodes with one child.

3. Yet even more partial TreeNode class and partial BinarySearchTree class.

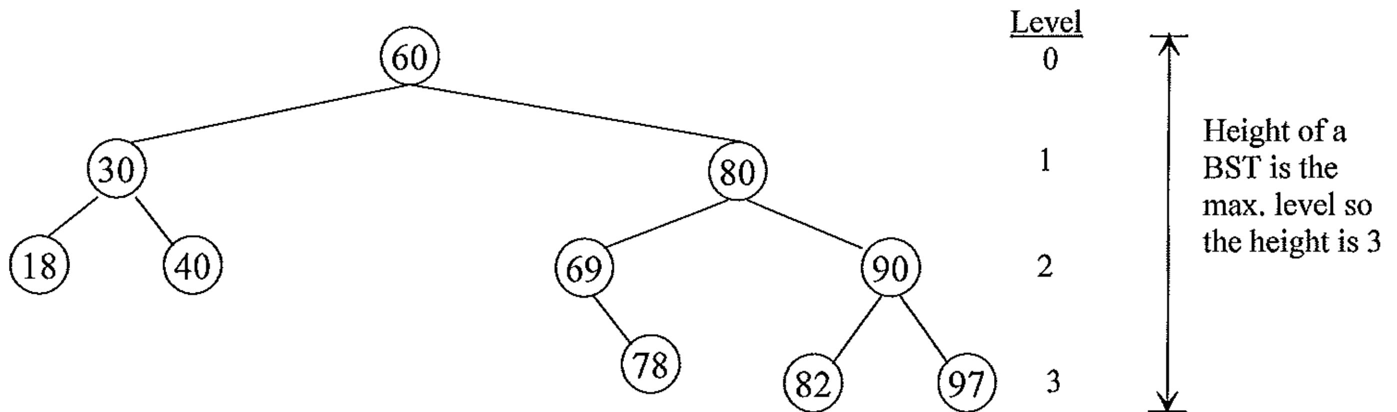
```
class TreeNode:
   . . .
      def findSuccessor(self):
         succ = None
         if self.hasRightChild():
             succ = self.rightChild.findMin()
         else:
             if self.parent:
                 if self.isLeftChild():
                     succ = self.parent
                 else:
                     self.parent.rightChild = None
                     succ = self.parent.findSuccessor()
                     self.parent.rightChild = self
         return succ

      def findMin(self):
         current = self
         while current.hasLeftChild():
             current = current.leftChild
         return current

      def spliceOut(self):
         if self.isLeaf():
            if self.isLeftChild():
               self.parent.leftChild = None
            else:
               self.parent.rightChild = None
         elif self.hasAnyChildren():
            if self.hasLeftChild():
               if self.isLeftChild():
                  self.parent.leftChild = self.leftChild
               else:
                  self.parent.rightChild = self.leftChild
               self.leftChild.parent = self.parent
            else:
               if self.isLeftChild():
                  self.parent.leftChild = self.rightChild
               else:
                  self.parent.rightChild = self.rightChild
               self.rightChild.parent = self.parent
```

Data Structures (CS 1520) Lab 9 Name:______________________

Objectives: You will gain experience BST performance and implementation

To start the lab: Download and unzip the file: http://www.cs.uni.edu/~fienup/cs1520f18/labs/lab9.zip

Part A: Consider the Binary Search Tree (BST) below. For each node in a BST, all values in the left-subtree are < the node and all values in the right-subtree are > the node.



a) Review section 6.5.2 on Tree Traversals to determine the order nodes are processed in each tree traversal.

- What is the order of node processing in a preorder traveral of the above BST?

- What is the order of node processing in a postorder traveral of the above BST?

- What is the order of node processing in a inorder traveral of the above BST?

b) Starting with an empty BST, what would be the shape of the BST after `put`'s for keys: 50, 60, 30, 70, 90, 40, 65?

After you have answered the above questions, raise your hand and explain your answers.

Part B: Run the `timeBinarySearchTree.py` program that:

- creates a list, evenList, that holds 5,000 sorted, even values (e.g., evenList = [0, 2, 4, 6, 8, ..., 9996, 9998])
- puts (adds) all the evenList items into an initially empty BinarySearchTree object, bst
- times the searches (in) bst for target values 0, 1, 2, 3, 4, ..., 9998, 9999 so half of the searches are successful and half are unsuccessful

a) How long does it take to search for target values of 0, 1, 2, 3, 4, ..., 9998, 9999?

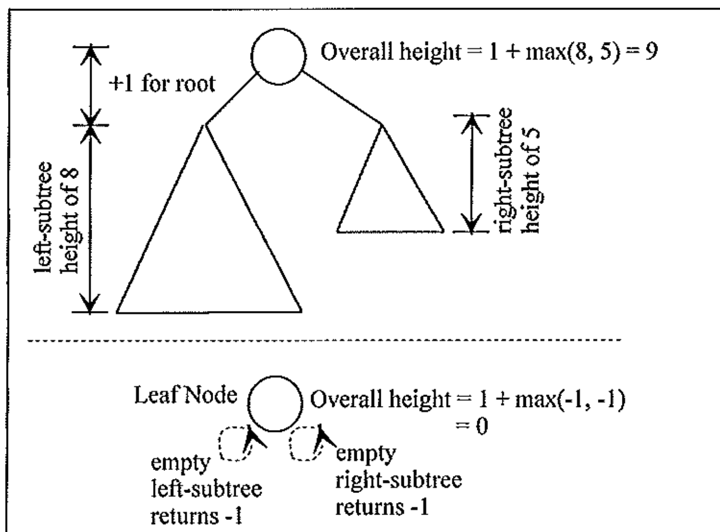
b) Explain why these searches take so long. (Hint: consider the shape of the BinarySearchTree `bst`)

c) Uncomment the "`shuffle(evenList)`" which randomizes the items in evenList before adding them to the BinarySearchTree `bst`. Now how long does it take to search for target values from 0, 1, 2, 3, 4, ..., 9998, 9999?

d) Explain why these searches take so little time.

e) What is the search time with the `timeOpenAddrHashDictSearch.py` program? Why is it faster?

Part C: a) **Complete the recursive `height` method in the BinarySearchTree class.** Model it after the postorder traversal, since the height of the whole BST can be determined after you know the height of the left-subtree and height of the right-subtree. For example if the left-subtree has a height of say 8 and the right-subtree has a height of 5, then the overall height including the root is 9 (i.e., one more than the tallest subtree's height). For the base case of the recursion, if we define the empty subtree's height to be -1 (i.e., `subtreeRoot` points to `None` since it has no `TreeNode` to point at), then the recursive definition still works for a leaf node which should have a height of 0.



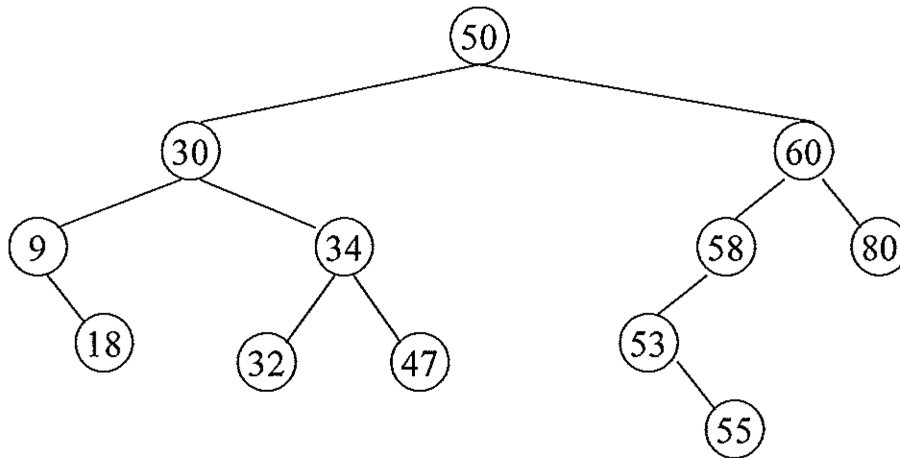
b) Uncomment the call to the `height` method at the end of the `timeBinarySearchTree.py` program. What is the height of `bst` if we are shuffling the `evenList`?

c) What would be the shortest possible height for a binary tree with 5,000 items?

After you have completed the height method and answered the above questions, raise your hand and explain your answers.

Data Structures (CS 1520) Lecture 20 Name:________________

1. Consider the Binary Search Tree (BST):



a. What would need to be done to delete 32 from the BST?

b. What would need to be done to delete 9 from the BST?

c. What would be the result of deleting 50 from the BST? Hint: One technique when programming is to convert a hard problem into a simpler problem. Deleting a BST node that contains two children is a hard problem. Since we know how to delete a BST node with none or one child, we can convert "deleting a node with two children" problem into a simpler problem by overwriting 50 with another node's value. Which nodes can be used to overwrite 50 and still maintain the BST ordering?

d. Which node would the `TreeNode's findSuccessor` method return for `succ` if we are deleting 50 from the BST?

2. When the `findSuccessor` method is called how many children does the `self` node have?

3. How could we improve the `findSuccessor` method?

4. When the `spliceOut` method is called from `remove` how many children could the `self` node have?

5. How could we improve the `spliceOut` method?

6. The shape of a BST depends on the order in which values are added (and deleted).

a) What would be the shape of a BST if we start with an empty BST and insert the sequence of values:

70, 90, 80, 5, 30, 110, 95, 40, 100

b) If a BST contains n nodes and we start searching at the root, what would be the worst-case big-oh *O*( ) notation for a successful search? (Draw the shape of the BST leading to the worst-case search)

7. We could store a BST in an array like we did for a binary heap, e.g. root at index 1, node at index i having left child at index 2 * i, and right child at index 2 * i + 1.

a) Draw the above BST (after inserting: 70, 90, 80, 5, 30, 110, 95, 40, 100) stored in an array (leave blank unused slots)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Index 0 Not Used																					

b) What would be the worst-case storage needed for a BST with n nodes?

8. a) If a BST contains n nodes, draw the shape of the BST leading to best, successful search in the worst case.

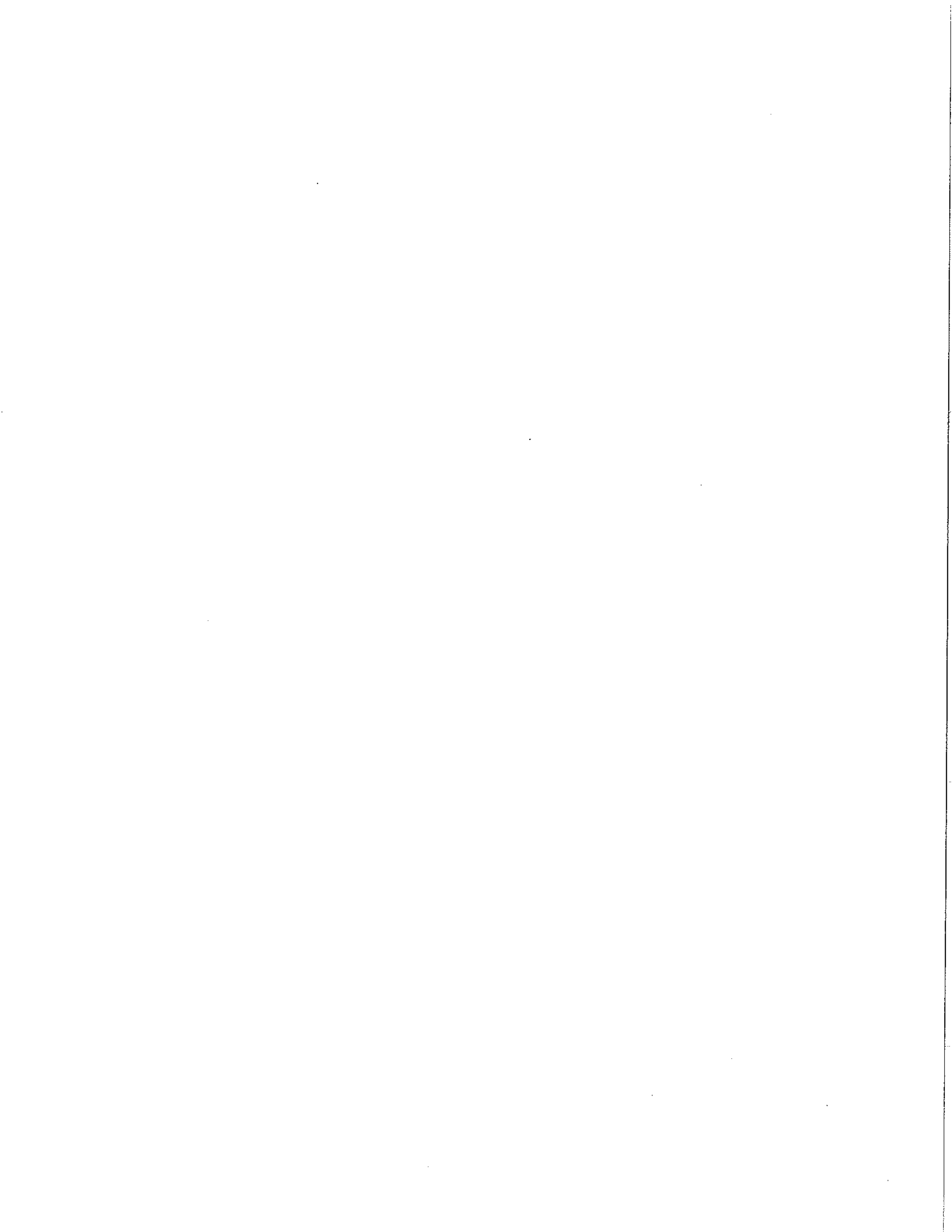
b) What is the worst-case big-oh *O*( ) notation for a successful search in this "best" shape BST?

Test 2 will be Thursday November 1st in class. It will be closed-book and notes, except for one 8.5" x 11" sheet of paper containing any notes that you want. (Yes, you can use both the front and back of this piece of paper.) Plus, you can use your Python Summary handout.

The test will cover Chapters 4 and 5. The following topics (and maybe more) with be covered:

Chapter 4: Recursion
Recursive functions: base-case(s), recursive case(s), tracing recursion via run-time stack or recursion tree, "infinite recursion"
Costs and benefits of recursion
Recursive examples: countDown, OrderedList __str__ method, fibonacci, factorial, binomial coefficient
Divide-and-Conquer technique of solving a problem. Examples: fibonacci, coin-change problem
Backtracking technique of solving a problem: Examples: coin-change problem, maze (textbook)
General concept of dynamic programming solutions for recursive problems that repeatedly solve the same smaller problems over and over. Example fibonacci, coin-change problem, binomial coefficient

Chapter 5: Searching and Sorting
Sequential/Linear search: code and big-oh analysis
Binary Search: code and big-oh analysis
Python List implementation (ListDict) of dictionaries and big-oh analysis
Hashing terminology: hash function, hash table, collision, load factor, chaining/closed-address/external chaining, open-address with some rehashing strategy: linear probing, quadratic probing, primary and secondary clustering
hashing implementation of dictionaries (ChainingDict and OpenAddrHashDict) and their big-oh analysis
General idea of simple sorts
Simple sorts: selection, bubble, insertion sorts and their big-oh analysis
Advanced sorts and their big-oh analysis: heap sort, quick sort and merge sort

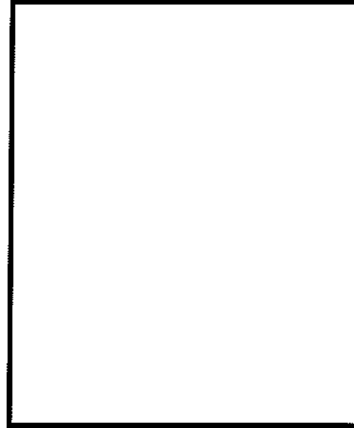


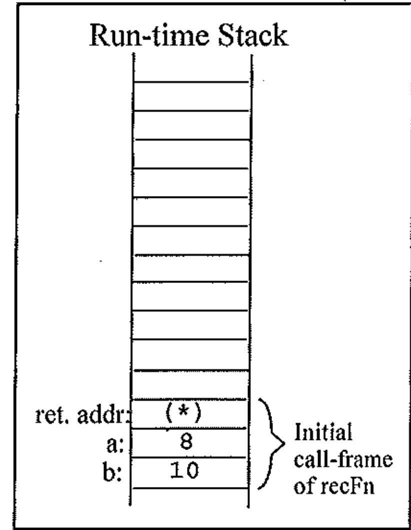
Spring 2018 Name: ____________________

Data Structures - Test 2

Question 1. (10 points) What is printed by the following program? **Output:**

```
def recFn(a, b):
    print( a, b )
    if  a < 0:
       return 100
    elif b < 0:
       return 1000
    elif a > b:     (**)
       return a + recFn(a - 3, b - 5)
   else:
       return recFn(a - 1, b - 3) - b
              (***)

print("Result = ", recFn(8, 10))
                    (*)
```



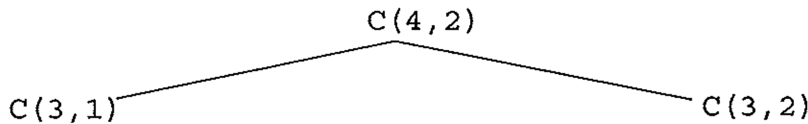


Question 2. a) (12 points) Write a recursive Python function to compute the binomial coefficent using the following recursive definition of $C(n, k)$:

$$C(n, k) = C(n-1, k-1) + C(n-1, k) \quad \text{for } 1 \leq k \leq (n-1), \text{ and}$$
$$C(n, k) = 1 \quad \text{for } k = 0 \text{ or } k = n$$

```
def C(n,k):
```

b) (8 points) For the above recursive function C(n,k), complete the calling-tree for C(4,2).



c) (3 points) What is the value of C(4,2)?

d) (2 points) What is the maximum number of call-frames of C on the run-time stack when calculating C(4,2) recursively?

Question 3. (15 points) Consider the following simple sorts discussed in class -- all of which sort in ascending order.

```
def bubbleSort(myList):
    for lastUnsortedIndex in range(len(myList)-1,0,-1):
        for testIndex in range(lastUnsortedIndex):
            if myList[testIndex] > myList[testIndex+1]:
                temp = myList[testIndex]
                myList[testIndex] = myList[testIndex+1]
                myList[testIndex+1] = temp
```

```
def insertionSort(myList):
    for firstUnsortedIndex in range(1,len(myList)):
        itemToInsert = myList[firstUnsortedIndex]
        testIndex = firstUnsortedIndex - 1
        while testIndex >= 0 and myList[testIndex] > itemToInsert:
            myList[testIndex+1] = myList[testIndex]
            testIndex = testIndex - 1

        myList[testIndex + 1] = itemToInsert
```

```
def selectionSort(aList):
    for lastUnsortedIndex in range(len(aList)-1, 0, -1):
        maxIndex = 0
        for testIndex in range(1, lastUnsortedIndex+1):
            if aList[testIndex] > aList[maxIndex]:
                maxIndex = testIndex
        # exchange the items at maxIndex and lastUnsortedIndex
        temp = aList[lastUnsortedIndex]
        aList[lastUnsortedIndex] = aList[maxIndex]
        aList[maxIndex] = temp
```

Timings of Above Sorting Algorithms on 10,000 items (seconds)			
Type of sorting algorithm	Initial Ordering of Items		
	Descending	Ascending	Random order
bubbleSort.py	23.3	7.7	15.8
insertionSort.py	14.2	0.004	7.3
selectionSort.py	7.3	7.7	6.8

a) Explain why bubbleSort on a descending list (23.3 s) takes longer than bubbleSort on an ascending list (7.7 s).

b) Explain why bubbleSort on a descending list (23.3 s) takes longer than insertionSort on a descending list (14.2 s).

c) Explain why selectionSort is $O(n^2)$ in the worst-case.

Spring 2018 Name: ____________________

Question 4. Two common rehashing strategies for open-address hashing are linear probing and quadratic probing:

quadratic probing	Check the square of the attempt-number away for an available slot, i.e., [home address + ((rehash attempt #)^2 +(rehash attempt #))//2] % (hash table size), where the hash table size is a power of 2. Integer division is used above

a) (8 points) Insert "Andrew Berns" **and then** "Sarah Diesburg" using Linear (on left) and Quadratic (on right) probing.

Hash Table with Linear Probing

0	Ben Schafer
1	
2	
3	Philip East
4	
5	
6	Mark Fienup
7	John Doe

Hash function

hash(John Doe) = 7
hash(Philip East) = 3
hash(Mark Fienup) = 6
hash(Ben Schafer) = 0
hash(Andrew Berns) = 7
hash(Sarah Diesburg) = 6

Hash Table with Quadratic Probing

0	Ben Schafer
1	
2	
3	Philip East
4	
5	
6	Mark Fienup
7	John Doe

b) In open-address hashing (like the pictures above), the average number probes/compares for various load factors is:

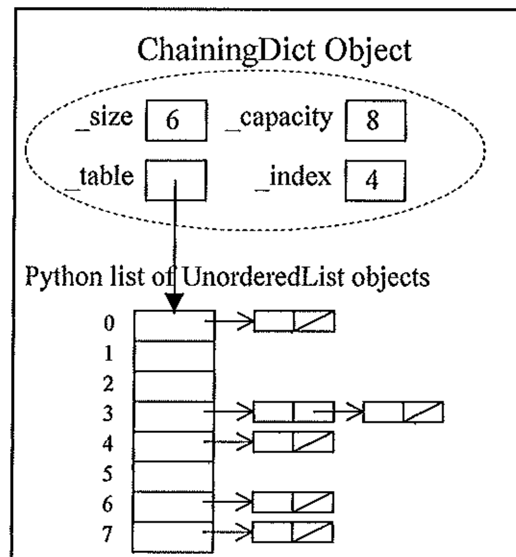
Probing Type	Search outcome	Load Factor, λ				
		0.25	**0.5**	**0.67**	**0.8**	**0.99**
Linear Probing	unsuccessful	1.39	2.50	5.09	13.00	5000.50
	successful	1.17	1.50	2.02	3.00	50.50
Quadratic Probing	unsuccessful	1.37	2.19	3.47	5.81	103.62
	successful	1.16	1.44	1.77	2.21	5.11

The "general rule of thumb" tries to keep the load factor (i.e., # items / hash-table size) between 0.5 and 0.67.

- (4 points) Why don't you want the load factor to exceed 0.67?

- (3 points) Why don't you want the load factor to be less than 0.5?

c) (5 points) In closed-address hashing (e.g., ChainingDict picture to the right) if the load factor (# items / hash table size) is 10, what would you expect for the average number of probes/compares of a successful search? (Justify your answer)



Question 5. (20 points) In class we discussed the bubbleSort code shown in question 3 on page 2 which sorts in ascending order (smallest to largest) and builds the sorted part on the right-hand side of the list.

For this question write a variation of bubble sort that:

- sorts in **descending order** (largest to smallest), and
- builds the **sorted part on the left-hand side** of the list, i.e.,

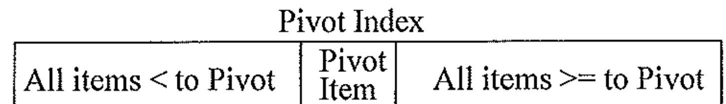
Inner loop scans from right to left across the unsorted part swapping adjacent items that are "out of order"

```
def bubbleSortVariation(myList):
```

Question 6. Recall the general idea of Quick sort:

- Partition by selecting a pivot item at "random" and then rearrange (*partitioning*) the unsorted items such that::
- Quick sort the unsorted part to the left of the pivot
- Quick sort the unsorted part to the right of the pivot

Pivot Index



a) (5 points) Explain why quick sort is $O(n \log_2 n)$ when sorting initially randomly ordered items. (n is the len(myList))

b) (5 points) Explain why quick sort is $O(n^2)$ is the worst-case. (n is the len(myList))