3. General "Algorithmic-Complexity Analysis" terminology:

- **problem**: question we seek an answer for, e.g., "What is the sum of all the items in a list/array?"
- **parameters**: variables with unspecified values
- **problem instance**: assignment of values to parameters, i.e., the specific input to the problem

```
myList: 0 1 2 3 4 5 6
```

(number of elements) \( n = 7 \)

**algorithm**: step-by-step procedure for producing a solution

**basic operation**: fundamental operation in the algorithm (i.e., operation done the most) Generally, we want to derive a function for the number of times that the basic operation is performed related to the **problem size**.

**problem size**: input size. For algorithms involving lists/arrays, the problem size is the number of elements ("n").

**Big-oh notation** \( O(\cdot) \): As the size of a problem grows (i.e., more data), how will our program's run-time grow.

Consider the following `sumList` function.

```
def sumList(myList):
    """Returns the sum of all items in myList""
    total = 0
    for item in myList:
        total = total + item
    return total
```

a) What is the basic operation of `sumList` (i.e., operation done the most)? **addition**

b) What is the problem size of `sumList`? **length of `myList`**

c) If \( n \) is 10000 and `sumList` takes 10 seconds, how long would you expect `sumList` to take for \( n \) of 20000?

d) What is the big-oh notation for `sumList`? \( O(n) \) ("linear time")

4. Consider the following `someLoops` function.

```
def someLoops(n):
    total = 0
    for i in range(n):
        for j in range(n):
            total = total + i + j
    return total
```

<table>
<thead>
<tr>
<th>Execution flow</th>
<th>i = 0</th>
<th>i = 1</th>
<th>i = 2</th>
<th>i = n - 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i = 0 ) to ( n-1 )</td>
<td>( j = 0 ) to ( n-1 )</td>
<td>( j = 0 ) to ( n-1 )</td>
<td>( j = 0 ) to ( n-1 )</td>
<td>loops ( n ) times</td>
</tr>
</tbody>
</table>

a) What is the basic operation of `someLoops` (i.e., operation done the most)? **addition**

b) How many times will the basic operation execute as a function of \( n \)? **\( n^2 \)**

c) What is the big-oh notation for `someLoops`? \( O(n^2) \) "quadratic"

d) If we input \( n \) of 10000 and `someLoops` takes 10 seconds, how long would you expect `someLoops` to take for \( n \) of 20000?

\[
T(n) = cn^2 + \text{const}
\]

\[
T(10000) = c \times 10000^2 \approx 1000000 \text{ sec}
\]

\[
T(20000) = c \times 20000^2 \approx 4000000 \text{ sec}
\]

\[
T(20000) = \frac{c \times 20000^2}{10000^2} = \left(\frac{10 \text{ sec}}{10000^2}\right)(20000)^2 = 4 \text{ sec}
\]
1. Draw the graph for `sumList (O(n))` and `someLoops (O(n^2))` from the previous lecture.

![Graph showing execution time vs. n for different complexities](image)

2. Consider the following `sumSomeListItems` function.

```python
import time

def main():
    n = eval(input("Enter size of list: "))
    aList = list(range(1, n+1))
    start = time.perf_counter()  # time.clock() is deprecated
    sum = sumSomeListItems(aList)
    end = time.perf_counter()
    print("Time to sum the list was {} seconds".format(end-start))

def sumSomeListItems(myList):
    """Returns the sum of some items in myList\n    """
    total = 0
    index = len(myList) - 1
    while index > 0:
        total = total + myList[index]
        index = (index // 2)
    return total

main()
```

a) What is the problem size of `sumSomeListItems`? \( \text{len(myList)} = n \)

b) If we input \( n \) of 10,000 and `sumSomeListItems` takes 10 seconds, how long would you expect `sumSomeListItems` to take for \( n \) of 20,000?

(Hint: For \( n \) of 20,000, how many more times would the loop execute than for \( n \) of 10,000?)

\[
T(n) = \frac{1}{10} \log_2 n
\]

\[
T(10000) = \frac{1}{10} \log_2 10000 = 10 \text{ sec}
\]

\[
T(20000) = \frac{1}{10} \log_2 20000
\]

\[
C = \frac{10 \text{ sec}}{13 \times 10^3}
\]

\[
C = \frac{10 \text{ sec}}{13 \times 10^3} (14 \times X)
\]

\[
\log_2 X = \frac{1}{2} \text{ iff } 2^{1/2} = X
\]

c) What is the big-oh notation for `sumSomeListItems`? \( \Theta(n) \)

d) Add the execution-time graph for `sumSomeListItems` to the graph.

```
3. 

\[ i = 1 \]

while \( i \leq n \):

for \( j \) in range(n):

\( \text{something of } O(1) \) \( \text{times} \)

\# end for

\( i = i \times 2 \)

\# end while

a) Analyze the above algorithm to determine its big-oh notation, \( O() \).

\[ O\left(n, \log_2 n\right) \]

b) If \( n \) of 10,000, takes 10 seconds, how long would you expect the above code to take for \( n \) of 20,000?

\[ T(n) = c \cdot n \log_2 n \]

\[ T(10000) = c \cdot 10000 \cdot \log_2 10000 = 10 \text{ sec} \]

\[ \frac{c}{10 \text{ sec} \cdot \log_2 10000} = \frac{10 \text{ sec}}{10000 \cdot \log_2 10000} \]

\[ c = 200000 \cdot \log_2 20000 \]

\[ T(20000) = \frac{10 \text{ sec}}{20000 \cdot \log_2 20000} \cdot 20000 \cdot \log_2 20000 \]

\[ = 20 \times 5 \text{ sec} \]

c) Add the execution-time graph for the above code to the graph.

4. Most programming languages have a built-in array data structure to store a collection of same-type items. Arrays are implemented in RAM memory as a contiguous block of memory locations. Consider an array \( X \) that contains the odd integers:

<table>
<thead>
<tr>
<th>address</th>
<th>Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>X[0]</td>
</tr>
<tr>
<td>4004</td>
<td>X[1]</td>
</tr>
<tr>
<td>4008</td>
<td>X[2]</td>
</tr>
<tr>
<td>4012</td>
<td>X[3]</td>
</tr>
<tr>
<td>4016</td>
<td>X[4]</td>
</tr>
<tr>
<td>4020</td>
<td>X[5]</td>
</tr>
<tr>
<td>4024</td>
<td>X[6]</td>
</tr>
</tbody>
</table>

a) Any array element can be accessed randomly by calculating its address. For example, address of \( X[5] = 4000 + 5 \times 4 = 4020 \). What is the general formula for calculating the address of the \( i \)th element in an array?

\[ X[i] = \left( \frac{5 \times i + 1}{\text{address}} \right) + \lambda \times \left( \frac{\text{item size}}{\text{bytes}} \right) \]

b) What is the big-oh notation for accessing the \( i \)th element?

\[ O(1) \text{ "constant time"} \]

c) A Python list uses an array of references (pointers) to list items in their implementation of a list. For example, a list of strings containing the alphabet:

\[ \text{mylist} = ['a', 'b', 'c', \ldots] \]

Since a Python list can contain heterogeneous data, how does storing references in the list aide implementation?

\[ \text{mylist}[i] \text{ constant time access since pointers are all the same size} \]
5. Arrays in most HLLs are static in size (i.e., cannot grow at run-time), so arrays are constructed to hold the "maximum" number of items. For example, an array with 1,000 slots might only contain 3 items:

<table>
<thead>
<tr>
<th>size</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>999</th>
</tr>
</thead>
<tbody>
<tr>
<td>scores</td>
<td>20</td>
<td>10</td>
<td>30</td>
<td>50</td>
<td>-</td>
</tr>
</tbody>
</table>

a) The physical size of the array is the number of slots in the array. What is the physical size of scores? 1000
b) The logical size of the array is the number of items actually in the array. What is the logical size of scores? 3

c) The load factor is fraction of the array being used. What is the load factor of scores? 0.03

d) What is the \(O()\) for appending a new score to the "right end" of the array? \(O(1)\)

e) What is the \(O()\) for adding a new score to the "left end" of the array? \(O(n)\)

f) What is the average \(O()\) for adding a new score to the array? \(O(\frac{1}{2}) = O(1)\)

g) During run-time if an array fills up and we want to add another item, the program can usually:
   - Create a bigger array than the one that filled up
   - Copy all the items from the old array to the bigger array
   - Add the new item
   - Delete the smaller array to free up its memory

When creating the bigger array, how much bigger than the old array should it be? Double physical size

h) What is the \(O()\) of moving to a larger array?

6. Consider the following list methods in Python:

<table>
<thead>
<tr>
<th>Method</th>
<th>Usage</th>
<th>Average (O()) for myList containing (n) items</th>
</tr>
</thead>
<tbody>
<tr>
<td>index</td>
<td>itemValue = myList[i]</td>
<td>(O(1))</td>
</tr>
<tr>
<td></td>
<td>myList[i] = newValue</td>
<td>(O(1))</td>
</tr>
<tr>
<td>append</td>
<td>myList.append(item)</td>
<td>(O(\frac{1}{2}))</td>
</tr>
<tr>
<td>extend</td>
<td>myList.extend(otherList)</td>
<td>(O(n))</td>
</tr>
<tr>
<td>insert</td>
<td>myList.insert(i, item)</td>
<td>(O(n))</td>
</tr>
<tr>
<td>pop</td>
<td>myList.pop()</td>
<td>(O(1))</td>
</tr>
<tr>
<td>pop(i)</td>
<td>myList.pop(i)</td>
<td>(O(1))</td>
</tr>
<tr>
<td>del</td>
<td>del myList[i]</td>
<td>(O(1))</td>
</tr>
<tr>
<td>remove</td>
<td>myList.remove(item)</td>
<td>(O(n))</td>
</tr>
<tr>
<td>.index</td>
<td>myList.index(item)</td>
<td>(O(n))</td>
</tr>
<tr>
<td>iteration</td>
<td>for item in myList:</td>
<td>(O(n))</td>
</tr>
<tr>
<td>reverse</td>
<td>myList.reverse()</td>
<td>(O(n))</td>
</tr>
</tbody>
</table>

Dictionary Operations:

<table>
<thead>
<tr>
<th>Method</th>
<th>Usage</th>
<th>Explanation</th>
<th>Average (O()) for (n) keys</th>
</tr>
</thead>
<tbody>
<tr>
<td>get item</td>
<td>myDictionary.get(myKey)</td>
<td>Returns the value associated with myKey; otherwise None</td>
<td>(O(1))</td>
</tr>
<tr>
<td>set item</td>
<td>myDictionary[myKey] = value</td>
<td>Change or add myKey: value pair</td>
<td>(O(1))</td>
</tr>
<tr>
<td>in</td>
<td>myKey in myDictionary</td>
<td>Returns True if myKey is in myDictionary; otherwise False</td>
<td>(O(1))</td>
</tr>
<tr>
<td>del</td>
<td>del myDictionary[myKey]</td>
<td>Deletes the mykey: value pair</td>
<td>(O(1))</td>
</tr>
</tbody>
</table>