

1. A *recursive function* is one that calls itself. Complete the recursive code for the `countDown` function that is passed a starting value and proceeds to count down to zero and prints “Blast Off!!!”.

Hint: The `countDown` function, like most recursive functions, solves a problem by splitting the problem into one or more simpler problems of the same type. For example, `countDown(10)` prints the first value (i.e, 10) and then solves the simpler problem of counting down from 9. To prevent “infinite recursion”, if-statement(s) are used to check for trivial *base case*(s) of the problem that can be solved without recursion. Here, when we reach a `countDown(0)` problem we can just print “Blast Off!!!”.

```
"""
File:  countDown.py
"""

def main():
    start = input("Enter count down start: ")
    print "\nCount Down:"
    countDown(start)

def countDown(count):

main()
```

**Program Output:**

```
Enter count down start: 10

Count Down:
10
9
8
7
6
5
4
3
2
1
Blast Off!!!
```

a) Trace the function call `countDown(5)` on paper by drawing the run-time stack and showing the output.

b) What do you think will happen if your call `countDown(-1)`?

c) Why is there a limit on the depth of recursion?

2. Some mathematical concepts are defining by recursive definitions. One example is the Fibonacci series:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

After the second number, each number in the series is the sum of the two previous numbers. The Fibonacci series can be defined recursively as:

$$\text{Fib}_0 = 0$$

$$\text{Fib}_1 = 1$$

$$\text{Fib}_N = \text{Fib}_{N-1} + \text{Fib}_{N-2} \text{ for } N \geq 2.$$

a) Complete the recursive function: `def fib (n) :`

b) Draw the *call tree* for `fib(5)`.

c) On my office computer, the call to `fib(40)` takes 22 seconds, the call to `fib(41)` takes 35 seconds, and the call to `fib(42)` takes 56 seconds. How long would you expect `fib(43)` to take?

d) How long would you guess calculating `fib(100)` would take on my office computer?

e) Why do you suppose this recursive `fib` function is so slow?

f) What is the computational complexity?  $O(\quad)$

g) How might we speed up the calculation of the Fibonacci series?

A VERY POWERFUL concept in Computer Science is *dynamic programming*. Dynamic programming solutions eliminate the redundancy of divide-and-conquer algorithms by calculating the solutions to smaller problems first, storing their answers, and looking up their answers if later needed instead of recalculating them.

We can use a list to store the answers to smaller problems of the Fibonacci sequence.

To transform from the recursive view of the problem to the dynamic programming solution you can do the following steps:

- 1) Store the solution to smallest problems (i.e., the base cases) in a list
- 2) Loop (no recursion) from the base cases up to the biggest problem of interest. On each iteration of the loop we:
  - solve the next bigger problem by looking up the solution to previously solved smaller problem(s)
  - store the solution to this next bigger problem for later usage so we never have to recalculate it

a) Complete the dynamic programming code:

```
def fib(n):
    """Dynamic programming solution to find the nth number in the Fibonacci seq."""

    # List to hold the solutions to the smaller problems
    fibonacci = []

    # Step 1: Store base case solutions
    fibonacci.append( )
    fibonacci.append( )

    # Step 2: Loop from base cases to biggest problem of interest
    for position in xrange( ):

        fibonacci.append( )

    # return nth number in the Fibonacci sequence
    return
```

Running the above code to calculate fib(100) would only take a fraction of a second.

b) One tradeoff of simple dynamic programming implementations is that they can require more memory since we store solutions to **all** smaller problems. Often, we can reduce the amount of storage needed if the next larger problem (and all the larger problems) don't really need the solution to the really small problems, but just the larger of the smaller problems. In fibonacci for example, when calculating the next value in the sequence we only need the previous two solution. Reimplement the `fib(n)` function a third time to eliminate the list `fibonacci` and replace it with three variables.