

Basic Data Structures

Stacks

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Outline

- 1 Stacks
 - What is a Stack?
 - The Stack Abstract Data Type
 - Implementing a Stack in Python
 - Simple Balanced Parentheses
 - Balanced Symbols (A General Case)
 - Converting Decimal Numbers to Binary Numbers
 - Infix, Prefix and Postfix Expressions

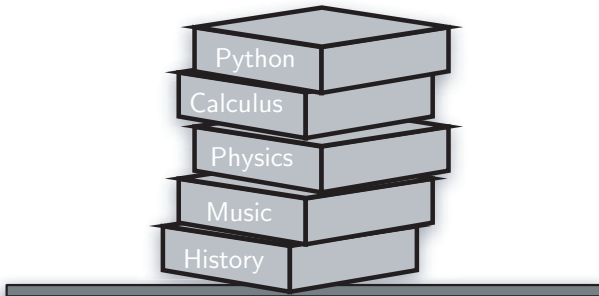
Outline

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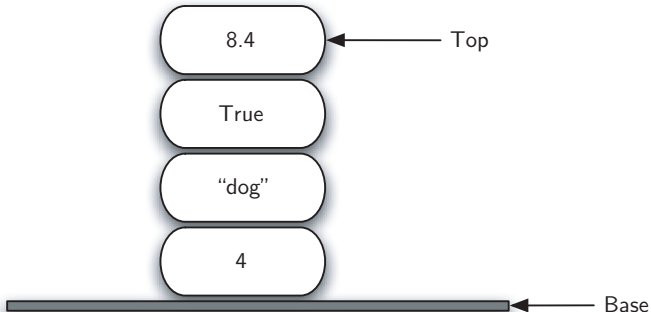
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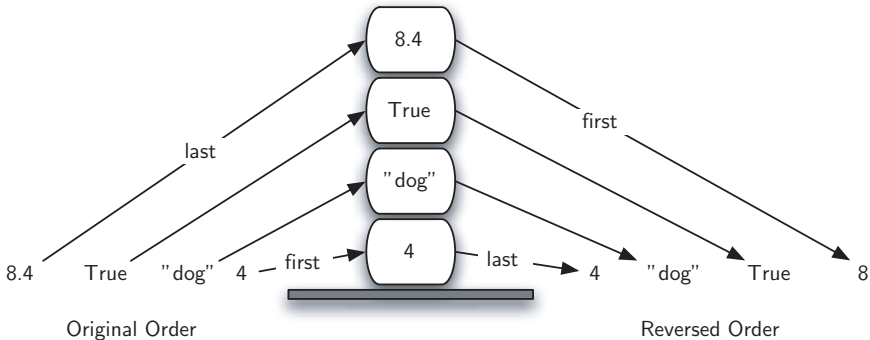
A Stack of Books



A Stack of Primitive Python Objects



The Reversal Property of Stacks



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- `Stack()` creates a new stack that is empty. It needs no parameters and returns an empty stack.
- `push(item)` adds a new item to the top of the stack. It needs the item and returns nothing.
- `pop()` removes the top item from the stack. It needs no parameters and returns the item. The stack is modified.
- `peek()` returns the top item from the stack but does not remove it. It needs no parameters. The stack is not modified.
- `isEmpty()` tests to see whether the stack is empty. It needs no parameters and returns a boolean value.
- `size()` returns the number of items on the stack. It needs no parameters and returns an integer.

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Stack Implementation in Python I

```
1  class Stack:
2      def __init__(self):
3          self.items = []
4
5      def isEmpty(self):
6          return self.items == []
7
8      def push(self, item):
9          self.items.append(item)
10
11     def pop(self):
12         return self.items.pop()
13
14
```

Stack Implementation in Python II

```
15     def peek(self):  
16         return self.items[len(self.items)-1]  
17  
18     def size(self):  
19         return len(self.items)
```

Alternative ADT Stack Implementation in Python I

```
1  class Stack:
2      def __init__(self):
3          self.items = []
4
5      def isEmpty(self):
6          return self.items == []
7
8      def push(self, item):
9          self.items.insert(0,item)
10
11     def pop(self):
12         return self.items.pop(0)
13
14
```

Alternative ADT Stack Implementation in Python II

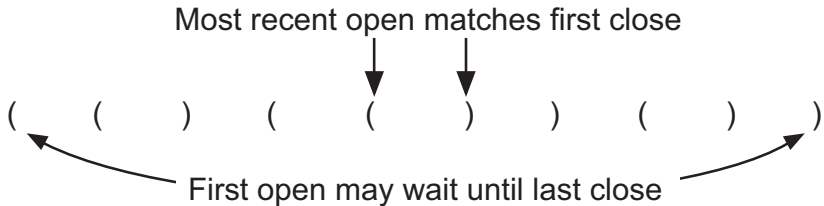
```
15     def peek(self):
16         return self.items[0]
17
18     def size(self):
19         return len(self.items)
```

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Matching Parentheses



Simple Balanced Parentheses I

```
1  def parChecker(symbolString):
2      s = Stack()
3
4      balanced = True
5      index = 0
6
7      while index < len(symbolString) and balanced:
8          symbol = symbolString[index]
9          if symbol == "(":
10             s.push(symbol)
11         else:
12
13
14
```


Simple Balanced Parentheses II

```
15         if s.isEmpty():
16             balanced = False
17         else:
18             s.pop()
19
20     index = index + 1
21
22 if balanced and s.isEmpty():
23     return True
24 else:
25     return False
```

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Balanced Symbols—A General Case I

```
1  def parChecker(symbolString):
2
3      s = Stack()
4
5      balanced = True
6      index = 0
7
8      while index < len(symbolString) and balanced:
9          symbol = symbolString[index]
10         if symbol in "([{":
11             s.push(symbol)
12         else:
13
14
```

Balanced Symbols—A General Case II

```
15
16         if s.isEmpty():
17             balanced = False
18         else:
19             top = s.pop()
20             if not matches(top, symbol):
21                 balanced = False
22
23         index = index + 1
24
25     if balanced and s.isEmpty():
26         return True
27     else:
28         return False
29
```

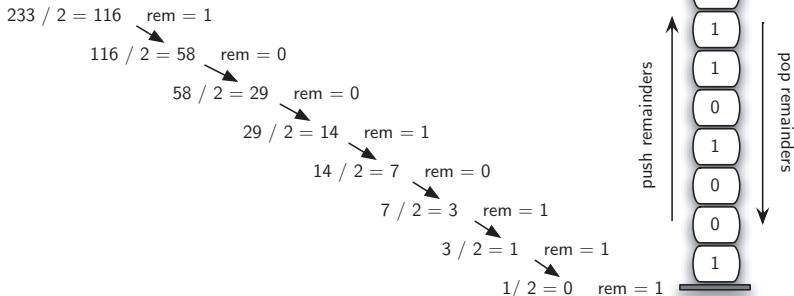
Balanced Symbols—A General Case III

```
30 def matches(open, close):
31     opens = "([{"
32     closers = ")]}"
33
34     return opens.index(open) == closers.index(close)
```

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Decimal-to-Binary Conversion



Decimal to Binary Conversion

```
1  def divideBy2(decNumber):
2
3      remstack = Stack()
4
5      while decNumber > 0:
6          rem = decNumber % 2
7          remstack.push(rem)
8          decNumber = decNumber / 2
9
10     binString = ""
11     while not remstack.isEmpty():
12         binString = binString + str(remstack.pop())
13
14     return binString
```


Conversion to Any Base I

```
1  def baseConverter(decNumber, base):
2
3      digits = "0123456789ABCDEF"
4
5      remstack = Stack()
6
7      while decNumber > 0:
8          rem = decNumber % base
9          remstack.push(rem)
10         decNumber = decNumber / base
11
12
13
14
```

Conversion to Any Base II

```
15     newString = ""
16     while not remstack.isEmpty():
17         newString = newString + digits[remstack.pop()]
18
19     return newString
```

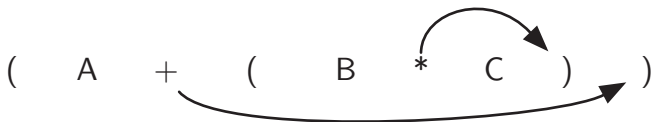
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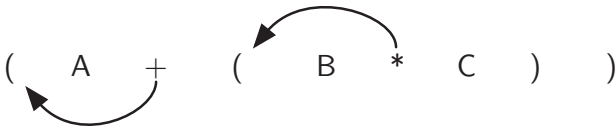
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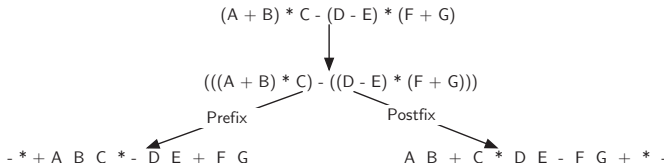
Moving Operators to the Right for Postfix Notation



Moving Operators to the Left for Prefix Notation



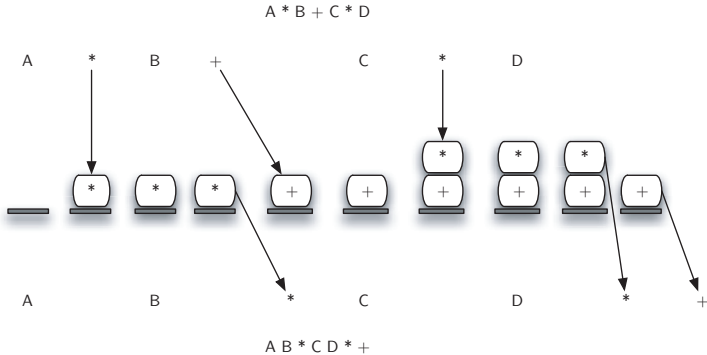
Converting a Complex Expression to Prefix and Postfix Notations



- 1 Create an empty stack called `opstack` for keeping operators. Create an empty list for output.
- 2 Convert the input infix string to a list by using the string method `split`.

- 3 Scan the token list from left to right.
 - If the token is an operand, append it to the end of the output list.
 - If the token is a left parenthesis, push it on the `opstack`.
 - If the token is a right parenthesis, pop the `opstack` until the corresponding left parenthesis is removed. Append each operator to the end of the output list.
 - If the token is an operator, `*`, `/`, `+`, or `-`, push it on the `opstack`. However, first remove any operators already on the `opstack` that have higher or equal precedence and append them to the output list.
- 4 When the input expression has been completely processed, check the `opstack`. Any operators still on the stack can be removed and appended to the end of the output list.

Converting $A * B + C * D$ to Postfix Notation



Converting Infix Expressions to Postfix Expressions I

```
1 import string
2 def infixToPostfix(infixexpr):
3
4     prec = {}
5     prec["*"] = 3
6     prec["/"] = 3
7     prec["+"] = 2
8     prec["-"] = 2
9     prec["("] = 1
10
11     opStack = Stack()
12     postfixList = []
13
14     tokenList = infixexpr.split()
```

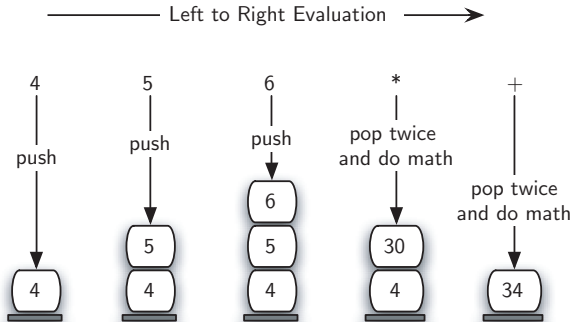
Converting Infix Expressions to Postfix Expressions II

```
15
16     for token in tokenList:
17         if token in string.uppercase:
18             postfixList.append(token)
19         elif token == '(' :
20             opStack.push(token)
21         elif token == ')' :
22             topToken = opStack.pop()
23             while topToken != '(' :
24                 postfixList.append(topToken)
25                 topToken = opStack.pop()
26
27
28
29
```

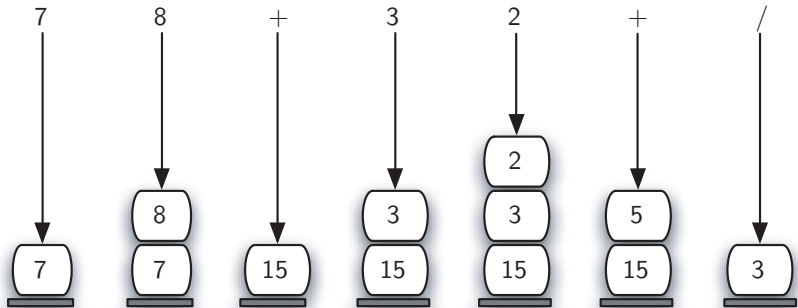
Converting Infix Expressions to Postfix Expressions III

```
30     else :
31         while (not opStack.isEmpty()) and \
32             (prec[opStack.peek()] >= prec[token]) :
33             postfixList.append(opStack.pop())
34
35         opStack.push(token)
36
37 while not opStack.isEmpty() :
38     postfixList.append(opStack.pop())
39
40 return string.join(postfixList)
```

Stack Contents During Evaluation



A More Complex Example of Evaluation



- 1 Create an empty stack called `operandStack`.
- 2 Convert the string to a list by using the string method `split`.
- 3 Scan the token list from left to right.
 - If the token is an operand, convert it from a string to an integer and push the value onto the `operandStack`.
 - If the token is an operator, `*`, `/`, `+`, or `-`, it will need two operands. Pop the `operandStack` twice. The first pop is the second operand and the second pop is the first operand. Perform the arithmetic operation. Push the result back on the `operandStack`.
- 4 When the input expression has been completely processed, the result is on the stack. Pop the `operandStack` and return the value.

Postfix Evaluation I

```
1 def postfixEval(postfixExpr):
2     operandStack = Stack()
3     tokenList = postfixExpr.split()
4
5     for token in tokenList:
6         if token in "0123456789":
7             operandStack.push(int(token))
8         else:
9             operand2 = operandStack.pop()
10            operand1 = operandStack.pop()
11            result = doMath(token, operand1, operand2)
12            operandStack.push(result)
13
14    return operandStack.pop()
```


Postfix Evaluation II

```
15
16 def doMath(op, op1, op2):
17     if op == "*":
18         return op1 * op2
19     else:
20         if op == "/":
21             return op1 / op2
22         else:
23             if op == "+":
24                 return op1 + op2
25             else:
26                 return op1 - op2
```