Data Structures (CS 1520) Lecture 10 Name:_________________

1. Consider the coin-change problem: Given a set of coin types and an amount of change to be returned, determine the fewest number of coins for this amount of change.

a) What "greedy" algorithm would you use to solve this problem with US coin types of {1, 5, 10, 25, 50} and a change amount of 29-cents?

b) Do you get the correct solution if you use this algorithm for coin types of {1, 5, 10, 12, 25, 50} and a change amount of 29-cents?

2. One way to solve this problem in general is to use a divide-and-conquer algorithm. Recall the idea of **Divide-and-Conquer** algorithms.
Solve a problem by:
- dividing it into smaller problem(s) of the same kind
- solving the smaller problem(s) recursively
- use the solution(s) to the smaller problem(s) to solve the original problem

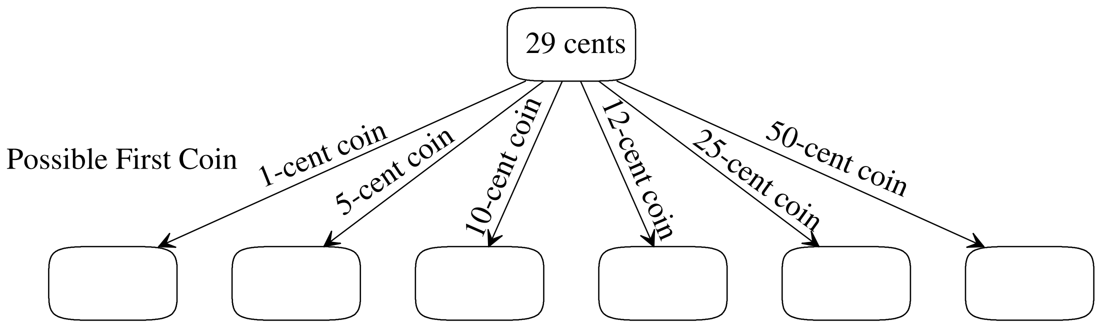
a) For the coin-change problem, what determines the size of the problem?

b) How could we divide the coin-change problem for 29-cents into smaller problems?

c) If we knew the solution to these smaller problems, how would be able to solve the original problem?

3. After we give back the first coin, which smaller amounts of change do we have?

Original Problem



Smaller problems

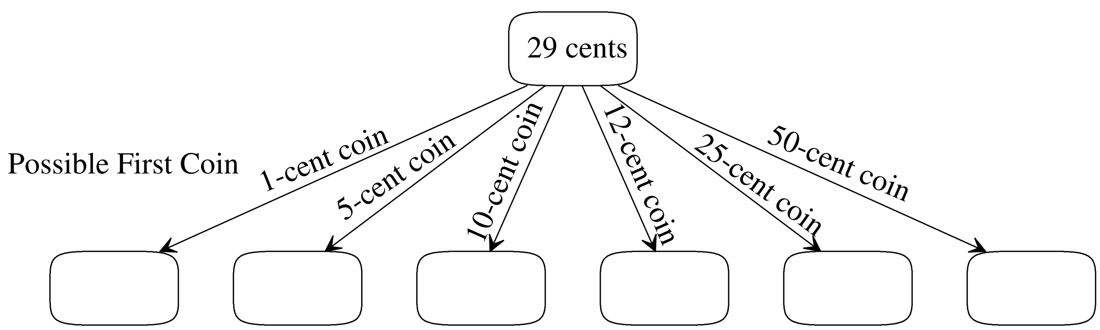
4. If we knew the fewest number of coins needed for each possible smaller problem, then how could determine the fewest number of coins needed for the original problem?

5. Complete a recursive relationship for the fewest number of coins.

$$\text{FewestCoins(change)} = \begin{cases} \min\limits_{\text{coin} \in \text{CoinSet and coin} \leq \text{change}}(\text{ FewestCoins(} \qquad\qquad\qquad \text{)) +} & \text{if change} \notin \text{ CoinSet} \\ 1 & \text{if change} \in \text{ CoinSet} \end{cases}$$

6. Complete a couple levels of the recursion tree for 29-cents change using the set of coins {1, 5, 10, 12, 25, 50}.

Original Problem



Smaller problems