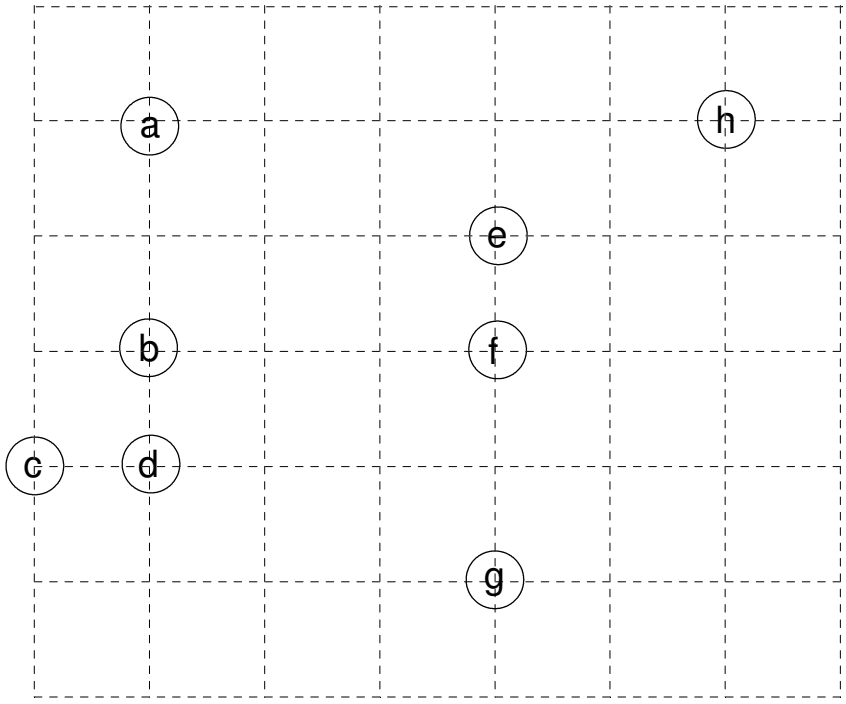
1. Suppose you had a map of settlements on the planet X
   (Assume edges could connecting all vertices with their Euclidean distances as their costs)



We want to build roads that allow us to travel between any pair of cities. Because resources are scarce, we want the total length of all roads build to be minimal. Since all cities will be connected anyway, it does not matter where we start, but assume we start at "a".

a) Assuming we start at city "a" which city would you connect first? Why this city?

b) What city would you connect next to expand your partial road network?

c) What would be some characteristics of the resulting "graph" after all the cities are connected?

d) Does your algorithm come up with the overall best (globally optimal) result?

2. Prim's algorithm for determining the minimum-spanning tree (MST) of a graph is another example of a *greedy algorithm*. Unlike divide-and-conquer and dynamic programming algorithms, greedy algorithms DO NOT divide a problem into smaller subproblems. Instead a greedy algorithm builds a solution by making a sequence of choices that look best ("locally" optimal) at the moment without regard for past or future choices (no backtracking to fix bad choices).

a) What greedy criteria does Prim's algorithm use to select the next vertex and edge to the partial minimum spanning tree?

b) Consider the textbook's Prim's Algorithm code (Listing 7.12 p. 346) which is incorrect.

```
def prim(G,start):
    pq = PriorityQueue()
    for v in G:
        v.setDistance(sys.maxsize)
        v.setPred(None)
    start.setDistance(0)
    pq.buildHeap([(v.getDistance(),v) for v in G])
    while not pq.isEmpty():
        currentVert = pq.delMin()
        for nextVert in currentVert.getConnections():
          newCost = currentVert.getWeight(nextVert) \
                    + currentVert.getDistance()
          if v in pq and newCost<nextVert.getDistance():
              nextVert.setPred(currentVert)
              nextVert.setDistance(newCost)
              pq.decreaseKey(nextVert,newCost)
```

c) What is wrong with the code? (Fix the above code.)

3. To avoid "massive" changes to the binHeap class, it can store `PriorityQueueEntry` objects:

```
class PriorityQueueEntry:
    def __init__(self,x,y):
        self.key = x
        self.val = y

    def getKey(self):
        return self.key

    def getValue(self):
        return self.val

    def setValue(self, newValue):
        self.val = newValue
```

```
    def __lt__(self,other):
        return self.key < other.key

    def __gt__(self,other):
        return self.key > other.key

    def __eq__(self, other):
        return self.val == other.val

    def __hash__(self):
        return self.key
```

a) Update the above Prim's algorithm code to use `PriorityQueueEntry` objects.
b) Why do the `__lt__` and `__gt__` methods compare `key` attributes, but `__eq__` compare `val` attributes?

c) When used for Prim's algorithm what type of objects are the `val`s compared by `__eq__`?

d) What changes to the Graph and Vertex classes need to be made?

e) Complete the `__contains__` and `decreaseKey` methods.

```
class BinHeap:
    def __init__(self):
        self.heapList = [0]
        self.currentSize = 0

    def buildHeap(self,alist):
        i = len(alist) // 2
        self.currentSize = len(alist)
        self.heapList = [0] + alist[:]
        while (i > 0):
            self.percDown(i)
            i = i - 1

    def percDown(self,i):
        while (i * 2) <= self.currentSize:
            mc = self.minChild(i)
            if self.heapList[i] > self.heapList[mc]:
                tmp = self.heapList[i]
                self.heapList[i] = self.heapList[mc]
                self.heapList[mc] = tmp
            i = mc

    def minChild(self,i):
        if i * 2 + 1 > self.currentSize:
            return i * 2
        else:
            if self.heapList[i*2] < self.heapList[i*2+1]:
                return i * 2
            else:
                return i * 2 + 1

    def percUp(self,i):
        while i // 2 > 0:
            if self.heapList[i] < self.heapList[i//2]:
               tmp = self.heapList[i // 2]
               self.heapList[i // 2] = self.heapList[i]
               self.heapList[i] = tmp
            i = i // 2

    def insert(self,k):
        self.heapList.append(k)
        self.currentSize = self.currentSize + 1
        self.percUp(self.currentSize)

    def delMin(self):
        retval = self.heapList[1]
        self.heapList[1] = self.heapList[self.currentSize]
        self.currentSize = self.currentSize - 1
        self.heapList.pop()
        self.percDown(1)
        return retval

    def isEmpty(self):
        return self.currentSize == 0

    def size(self):
        return self.currentSize

    def __str__(self):
        return str(self.heapList[1:])
```

```
def __contains__(self, value):
```

```
def decreaseKey(self, decreasedValue):
    """Precondition:  decreasedValue
       in heap already"""
```