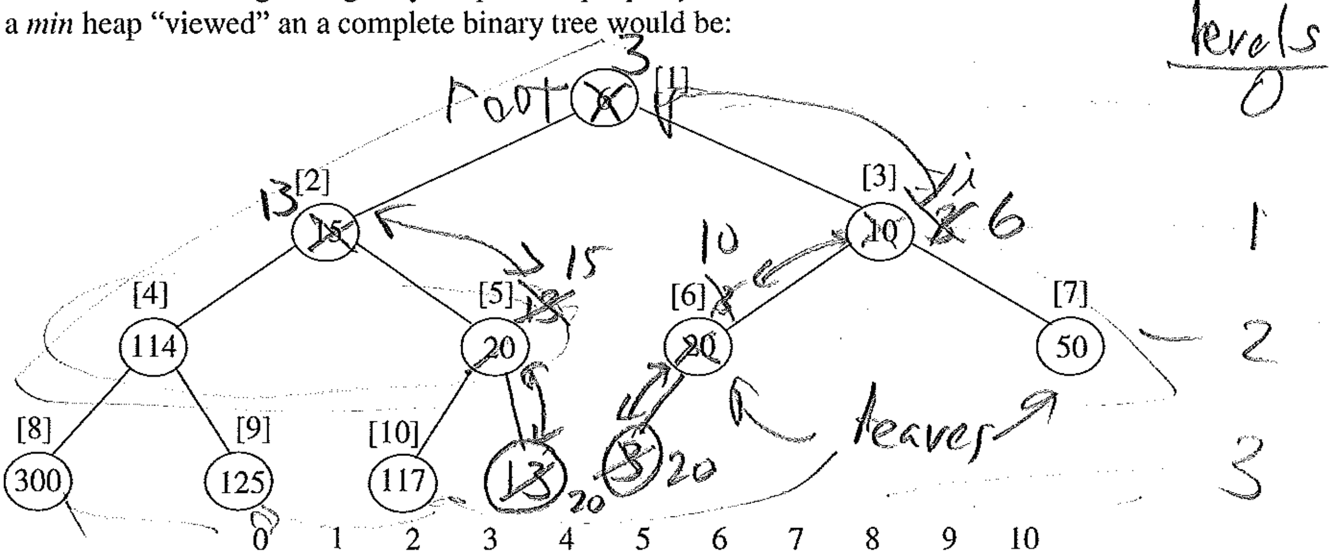
Data Structures (CS 1520) Lecture 7 Name: Mark F.

1. Section 6.6 discusses a very "non-intuitive", but powerful list/array-based approach to implement a priority queue, call a binary heap. The list/array is used to store a *complete binary tree* (a full tree with any additional leaves as far left as possible) with the items being arranges by *heap-order property*, i.e., each node is ≤ either of its children. An example of a *min* heap "viewed" an a complete binary tree would be:



Python List actually used to store heap items	0	1	2	3	4	5	6	7	8	9	10
	not used	6	15	10	114	20	20	50	300	125	117

a) For the above heap, the list/array indexes are indicated in [ ]'s. For a node at index *i*, what is the index of:

- its left child if it exists: i * 2
- its right child if it exists: i * 2 + 1
- its parent if it exists: i // 2

b) What would the above heap look like after inserting 13 and then 3? (show the changes on above tree)

General Idea of insert(newItem):

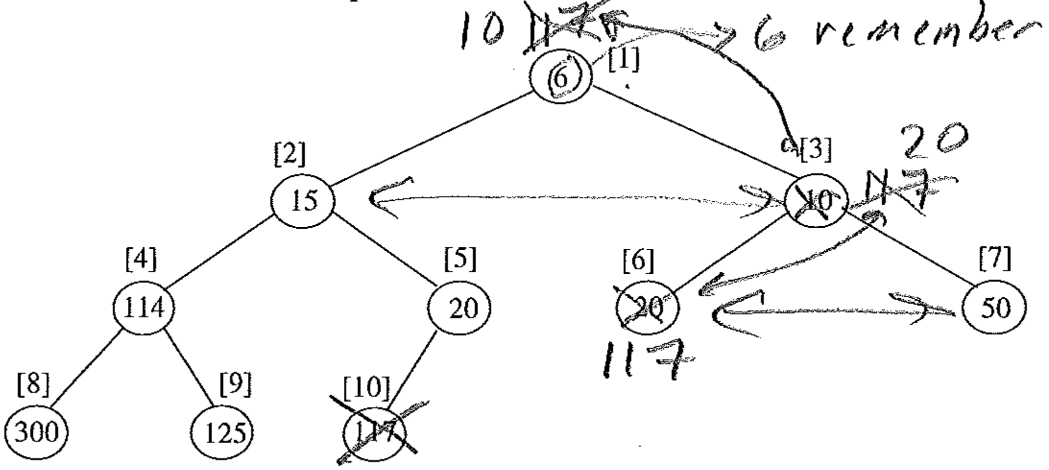
- append newItem to the end of the list (easy to do, but violates heap-order property)
- restore the heap-order property by repeatedly swapping the newItem with its parent until it *percolates* to correct spot

c) What is the big-oh notation for inserting a new item in the heap? $O(\log_2 n)$

d) Complete the code for the percUp method used by insert.

```
class BinHeap:
    def __init__(self):
        self.heapList = [0]
        self.currentSize = 0

    def percUp(self,currentIndex):
        parentIndex = CurrentIndex // 2
        while currentIndex > 1 and self.heapList[currentIndex]
                                     < self.heapList[parentIndex]:
            temp = self.heapList[currentIndex]
            self.heapList[currentIndex] = self.heapList[parentIndex]
            self.heapList[parentIndex] = temp
            currentIndex = parentIndex


    def insert(self,k):
        self.heapList.append(k)
        self.currentSize = self.currentSize + 1
        self.percUp(self.currentSize)
```

2. Now let us consider the `delMin` operation that removes and returns the minimum item.



Python List actually used to store heap items	0	1	2	3	4	5	6	7	8	9	10
	not used	6	15	10	114	20	20	50	300	125	117

a) What item would `delMin` remove and return from the above heap? 6

b) What is the quickest way to fill the hole left by `delMin`? move right-most item to overwrite it e.g. 117

c) What new problem does this cause? need to restore heap-order property

General Idea of `delMin()`:

- remember the minimum value so it can be returned later (easy to find - at index 1)
- copy the last item in the list to the root, delete it from the right end, decrement size
- restore the heap-order property by repeatedly swapping this item with its smallest child until it *percolates down* to the correct spot
- return the minimum value

d) What would the above heap look like after `delMin`? (show the changes on above tree)

e) Complete the code for the percDown method used by `delMin`.

```
class BinHeap:
              .
              .
              .

    def minChild(self,i):
       if i * 2 + 1 > self.currentSize: # if only left child
           return i * 2
       else:
           if self.heapList[i * 2] < self.heapList[i * 2 + 1]:
                return i * 2
           else:
                return i * 2 + 1

    def delMin(self):
       retval = self.heapList[1]
       self.heapList[1] = self.heapList[self.currentSize]
       self.currentSize = self.currentSize - 1
       self.heapList.pop()
       self.percDown(1)
       return retval
```

```
def percDown(self,currentIndex):
    while currentIndex*2>self.currentSize:
        minIndex = self.minChild(currentIndex)
        if self.heapList[currentIndex]
          >self.heapList[minIndex]:
            temp=self.heapList[currentIndex]
            self.heapList[currentIndex]
                = self.heapList[minIndex]
            self.heapList[minIndex]=temp
            currentIndex = minIndex
        else:
            break
```

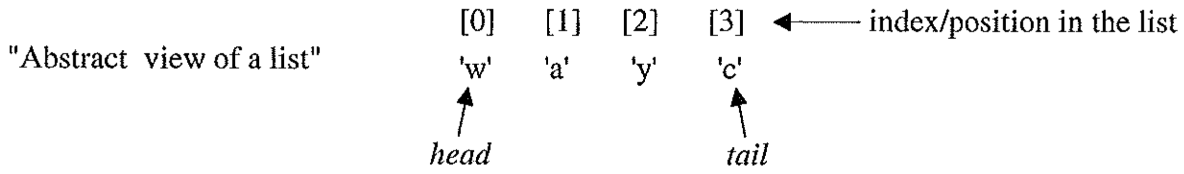
f) What is the big-oh notation for `delMin`?

Once we have a working `BinHeap`, then implementing the `PriorityQueue` class using a `BinHeap` is a piece of cake:

```
### File: priority_queue.py
from binheap import BinHeap

class PriorityQueue:
    def __init__(self):
        self._heap = BinHeap()

    def isEmpty(self):
        return self._heap.isEmpty()

    def enqueue(self, item):
        self._heap.insert(item)

    def dequeue(self):
        return self._heap.delMin()

    def size(self):
        return self._heap.size()

    def __str__(self):
        return str(self._heap)
```

```
>>> q = PriorityQueue()
>>> print(q)
[]
>>> q.enqueue(5)
>>> q.enqueue(1)
>>> q.enqueue(7)
>>> print(q)
[1, 5, 7]
>>> q.dequeue()
1
>>> print(q)
[5, 7]
```

3. A "list" is a generic term for a sequence of items in a linear arrangement. Unlike stacks, queues and deques access to list items is not limited to either end, but can be from any position in the list. The general terminology of a list is illustrated by:



There are three broad categories of list operations that are possible:

- **index-based operations** - the list is manipulated by specifying an index location, e.g.,
  myList.insert(3, item) # insert item at index 3 in myList
- **content-based operations** - the list is manipulated by specifying some content (i.e., item value), e.g.,
  myList.add(item) # adds the item to the list
- **cursor-base operations** - a *cursor* (current position) can be moved around the list, and it is used to identify list items to be manipulated, e.g.,
  myList.first() # sets the cursor to the head item of the list
  myList.next() # moves the cursor one position toward the tail of the list
  myList.remove() # deletes the second item in the list because that's where the cursor is currently located

The following table summarizes the operations from the three basic categories on a list, L:

Index-based operations	**Content-based operations**	**cursor-based operations**
`L.insert(index, item)`	`L.add(item)`	`L.hasNext()`
`item = L[index]`	`L.remove(item)`	`L.next()`
`L[index] = newValue`	`L.search(item) #return Boolean`	`L.hasPrevious()`
`L.pop(index)`	`i = L.index(item)`	`L.previous()`
		`L.first()`
		`L.last()`
`L.append(item)`		`L.insert(item)`
`L.pop()`		`L.replace(item)`
		`L.remove()`

Built-in Python lists are unordered with a mixture of index-based and content-based operations. We know they are implemented using a contiguous block of memory (i.e., an array). The textbook talks about an unordered list ADT, and a sorted list ADT which is more content-based. Both are implemented using a singly-linked list.

a) Why would a singly-linked list be a bad choice for implementing a cursor-based list ADT? Slow (O(n)) to perform previous method. Better to use doubly-linked list.