

Question 1. (4 points) Consider the following Python code.

```

i = n
while i > 1:
    for j in range(n * n):
        print(i, j)
    i = i // 2
  
```

$$O(n^2 \log_2 n)$$

What is the big-oh notation  $O()$  for this code segment in terms of  $n$ ?

Question 2. (4 points) Consider the following Python code.

```

for i in range(n):
    for j in range(n):
        print(j)
    for k in range(n):
        print(k)
  
```

$$O(n^2)$$

What is the big-oh notation  $O()$  for this code segment in terms of  $n$ ?

Question 3. (4 points) Consider the following Python code.

```

def main(n):
    for i in range(n):
        doSomething(n)
def doSomething(n):
    for k in range(n):
        doMore(n)
def doMore(n):
    for j in range(n):
        print(j)
main(n)
  
```

$$O(n^3)$$

What is the big-oh notation  $O()$  for this code segment in terms of  $n$ ?

Question 4. (8 points) Suppose a  $O(n^3)$  algorithm takes 10 second when  $n = 100$ . How long would the algorithm run when  $n = 1,000$ ?

$$T(n) = cn^3 \quad T(100) = c \cdot 100^3 = 10 \text{ sec} \quad \left| \quad T(1000) = c \cdot 1000^3 \right.$$

$$c = \frac{10 \text{ sec}}{100^3} = 10^{-5} \text{ sec} \quad \left| \quad = \frac{1000^3}{10^5} = \frac{10^9}{10^5} = 10^4 \text{ sec} \right.$$

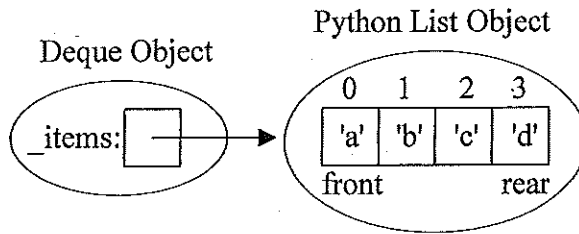
10,000 sec

Question 5. (10 points) Why should any method/function having a "precondition" raise an exception if the precondition is violated?

To inform the programmer that an error in using the function has occurred. It aids in debugging of the program since error detected immediately.

Question 6. A Deque (pronounced "Deck") is a linear data structure which behaves like a double-ended queue, i.e., it allows adding or removing items from either the front or the rear of the Deque. One possible implementation of a Deque would be to use a built-in Python list to store the Deque items such that

- the front item is always stored at index 0,
- the rear item is always at index  $\text{len}(\text{self.}_\text{items}) - 1$  or -1



a) (6 points) Complete the big-oh  $O()$ , for each Deque operation, assuming the above implementation. Let  $n$  be the number of items in the Deque.

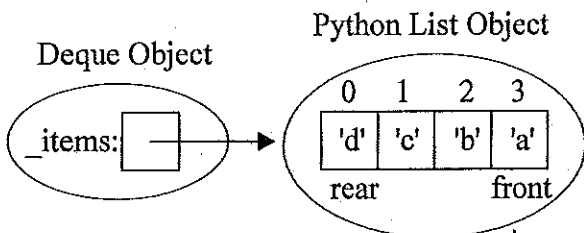
isEmpty	addRear	removeRear	addFront	removeFront	size
$O(1)$	$O(1)$	$O(1)$	$O(n)$	$O(n)$	$O(1)$

b) (9 points) Complete the method for the removeFront operation, including the precondition check to raise an exception if it is violated.

```
def removeFront(self):
    """Removes and returns the Front item of the Deque
    Precondition: the Deque is not empty.
    Postcondition: Front item is removed from the Deque and returned"""
```

```
if len(self._items) == 0:
    raise ValueError("cannot remove front from empty Deque")
return self._items.pop(0)
```

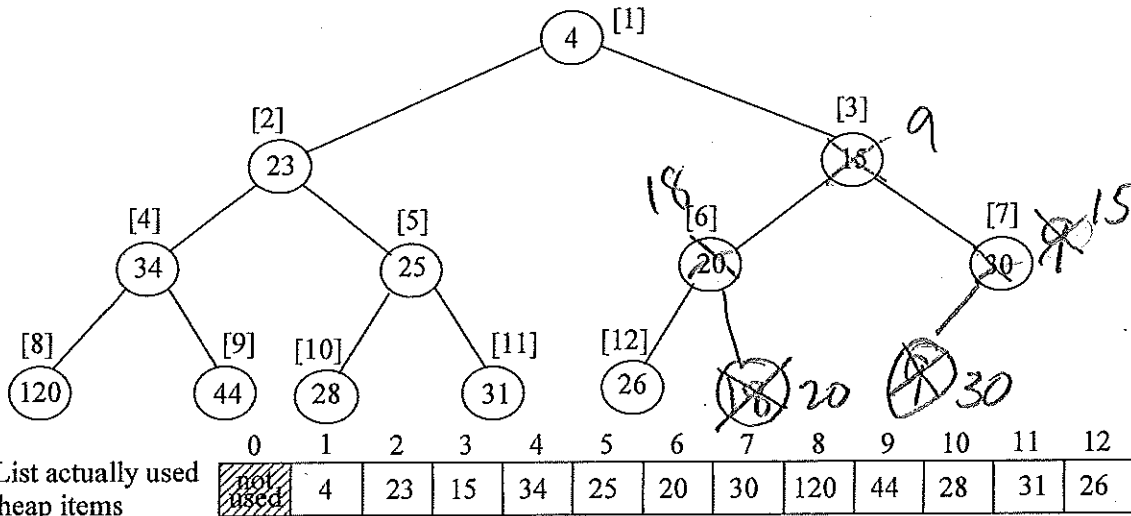
c) (5 points) An alternate Deque implementation would swap the location of the front and rear items as in:



Why is this alternate implementation probably not very helpful with respect to the Deque's performance?

addRear + removeRear become  $O(n)$   
and addFront and removeFront become  $O(1)$ ,  
so it just swaps the  $O(n)$ 's around.

Question 7. Consider the binary heap approach to implement a priority queue. A Python list is used to store a *complete binary tree* (a full tree with any additional leaves as far left as possible) with the items being arranged by *heap-order property*, i.e., each node is  $\leq$  either of its children. An example of a *min heap* "viewed" as a complete binary tree would be:



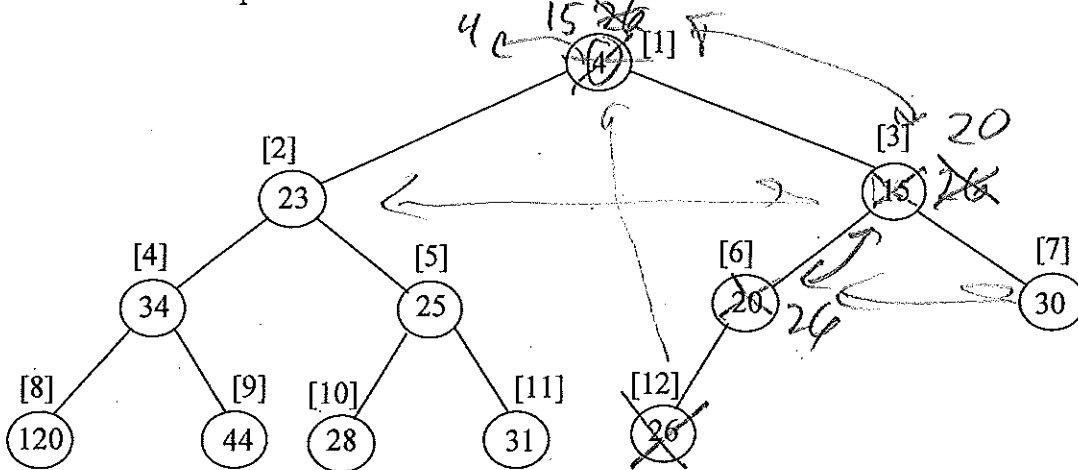
- a) (3 points) For the above heap, the list indexes are indicated in [ ]'s. For a node at index  $i$ , what is the index of:
- its left child if it exists:  $2 * i$
  - its right child if it exists:  $2 * i + 1$
  - its parent if it exists:  $i // 2$

b) (7 points) What would the above heap look like after inserting 18 and then 9 (show the changes on above tree)

c) (6 points) What is the big-oh notation for the insert operation? (EXPLAIN YOUR ANSWER)

$O(\log_2 n)$  The inserted it starts at index  $n$  where  $n$  is the # of items in the heap. Its index in the worst case is halved ( $i // 2$ ) until it reaches 1.

Now consider the `delMin` operation that removes and returns the minimum item.

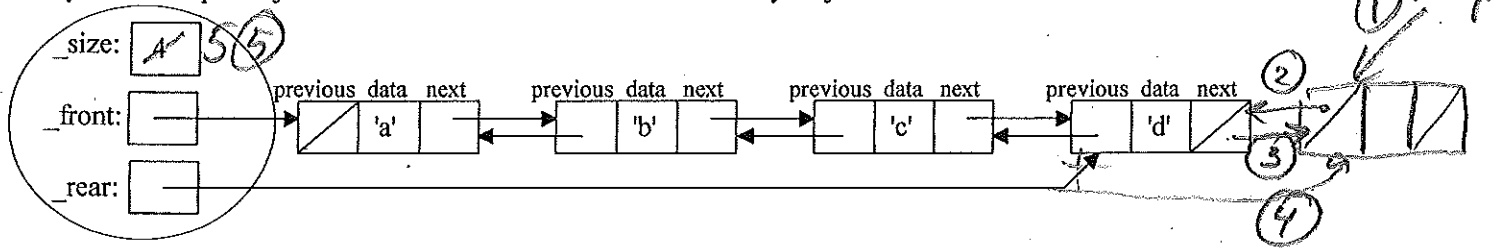


- d) (2 point) What item would `delMin` remove and return from the above heap? 4
- e) (7 points) What would the above heap look like after `delMin`? (show the changes on above tree)

Question 8. The Node2Way class (which inherits the node.py class) can be used to dynamically create storage for each new item added to a Deque using a doubly-linked implementation as in:

DoublyLinkedDeque Object

Node2Way Objects



a) (6 points) Determine the big-oh,  $O()$ , for each Deque operation assuming the above doubly-linked implementation. Let  $n$  be the number of items in the Deque.

addFront	removeFront	addRear	removeRear	size	<u>str</u>
$O(1)$	$O(1)$	$O(1)$	$O(1)$	$O(1)$	$O(n^2)$

b) (14 points) Complete the addRear method.

```
class DoublyLinkedDeque(object):
    """ Doubly-Linked list based Deque implementation. """
    def __init__(self):
        self._size = 0
        self._front = None
        self._rear = None
    def addRear(self, newItem):
        """ Adds the newItem to the rear of the Deque.
            Precondition: none """
        temp = Node2Way(newItem)
        if self._size == 0:
            self._front = temp
        else:
            temp.setPrevious(self._rear)
            self._rear.getNext(temp)
        self._rear = temp
        self._size += 1
```

```
class Node:
    def __init__(self, initdata):
        self.data = initdata
        self.next = None
    def getData(self):
        return self.data
    def getNext(self):
        return self.next
    def setData(self, newdata):
        self.data = newdata
    def setNext(self, newnext):
        self.next = newnext
```

```
class Node2Way(Node):
    def __init__(self, initdata):
        Node.__init__(self, initdata)
        self.previous = None
    def getPrevious(self):
        return self.previous
    def setPrevious(self, newprevious):
        self.previous = newprevious
```

Picture + 2  
# of steps + 2  
Normal case code + 7  
Special case(s) + 3

c) (5 points) Would using singly-linked nodes (i.e., Node objects instead of Node2Way) slow down any of the Deque operations? Justify your answer

Yes, the removeRear would become  $O(n)$  because we need to reset the self.\_rear pointer by starting at the first node and traversing down the chain of nodes