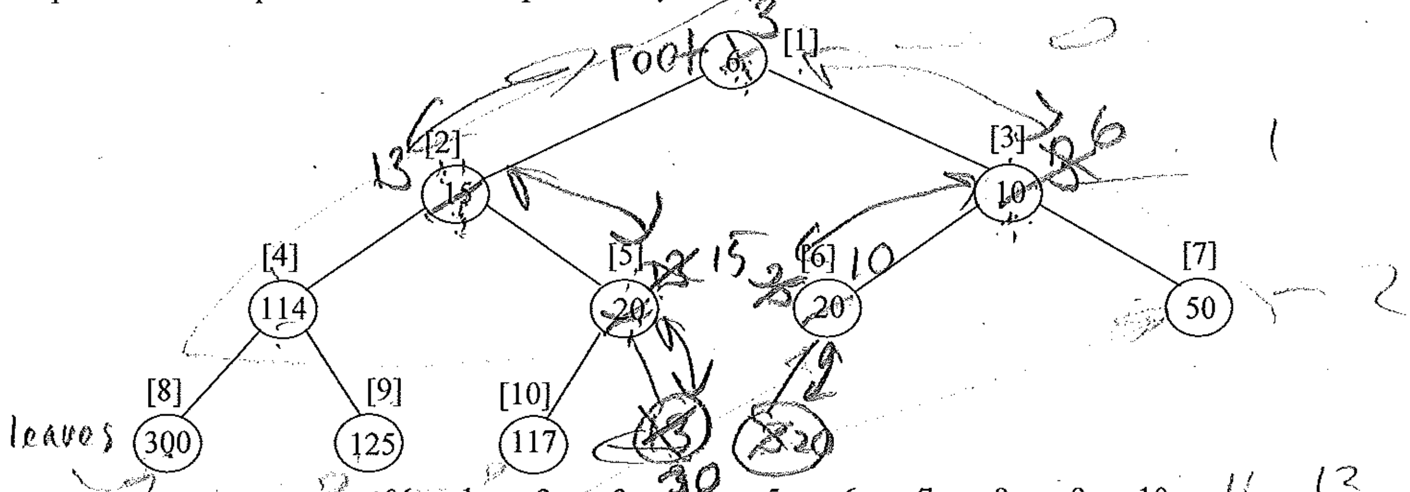
1. Section 6.6 discusses a very "non-intuitive", but powerful list/array-based approach to implement a priority queue, call a binary heap. The list/array is used to store a *complete binary tree* (a full tree with any additional leaves as far left as possible) with the items being arranges by *heap-order property*, i.e., each node is ≤ either of its children. An example of a *min* heap "viewed" an a complete binary tree would be:



Python List actually used to store heap items

0	1	2	3	4	5	6	7	8	9	10	11	12
not used	6	15	10	114	20	20	50	300	125	117	13	3

a) For the above heap, the list/array indexes are indicated in [ ]'s. For a node at index *i*, what is the index of:

- its left child if it exists: i * 2
- its right child if it exists: i * 2 + 1
- its parent if it exists: i // 2

b) What would the above heap look like after inserting 13 and then 3? (show the changes on above tree)

General Idea of insert(newItem):

- append newItem to the end of the list (easy to do, but violates heap-order property)
- restore the heap-order property by repeatedly swapping the newItem with its parent until it *percolates* to correct spot

c) What is the big-oh notation for inserting a new item in the heap? O(log₂ n)

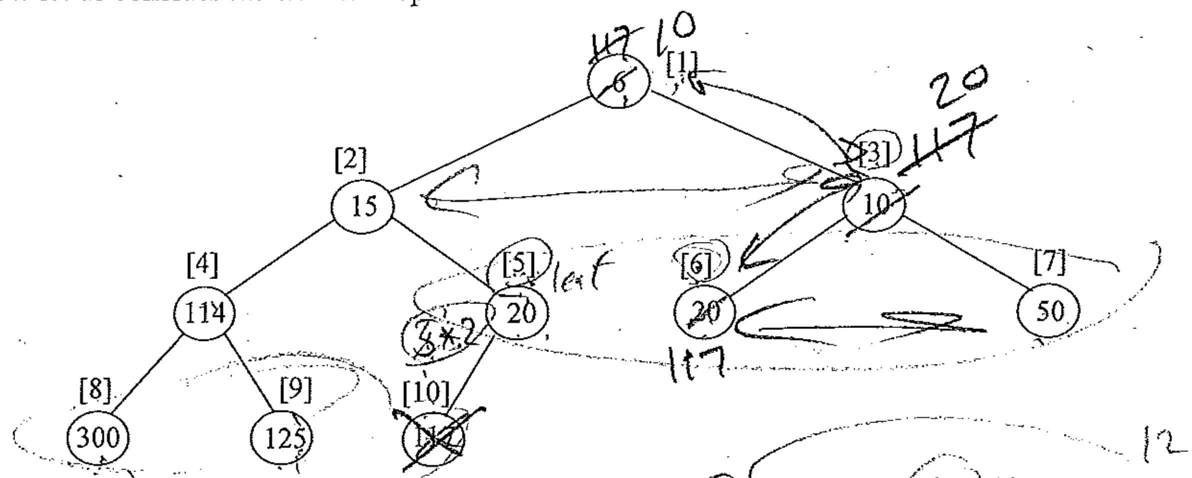
d) Complete the code for the percUp method used by insert.

```
class BinHeap:
    def __init__(self):
        self.heapList = [0]
        self.currentSize = 0

    def percUp(self,currentIndex):
        parentIndex = currentIndex // 2
        while currentIndex > 1 and self.heapList[currentIndex] < self.heapList[parentIndex]:
            temp = self.heapList[currentIndex]
            self.heapList[currentIndex] = self.heapList[parentIndex]
            self.heapList[parentIndex] = temp
            currentIndex = parentIndex
            parentIndex = currentIndex // 2

    def insert(self,k):
        self.heapList.append(k)
        self.currentSize = self.currentSize + 1
        self.percUp(self.currentSize)
```

2. Now let us consider the delMin operation that removes and returns the minimum item.



Python List actually used to store heap items	0	1	2	3	4	5	6	7	8	9	10
	not used	6	15	10	114	20	20	50	300	125	117

a) What item would delMin remove and return from the above heap? 6 at index 1

b) What is the quickest way to fill the hole left by delMin? grab last item to fill hole at index 1

c) What new problem does this cause?

General Idea of delMin():

- remember the minimum value so it can be returned later (easy to find - at index 1)
- copy the last item in the list to the root, delete it from the right end, decrement size
- restore the heap-order property by repeatedly swapping this item with its smallest child until it *percolates down* to the correct spot
- return the minimum value

d) What would the above heap look like after delMin? (show the changes on above tree)

e) Complete the code for the percDown method used by delMin.

```
class BinHeap:
        .
        .
        .
    def minChild(self,i):          # precond: index i is not a leaf
        if i * 2 + 1 > self.currentSize: # if only left child
            return i * 2
        else:
            if self.heapList[i * 2] < self.heapList[i * 2 + 1]:
                return i * 2
            else:
                return i * 2 + 1

    def delMin(self):
        retval = self.heapList[1]
        self.heapList[1] = self.heapList[self.currentSize]
        self.currentSize = self.currentSize - 1
        self.heapList.pop()
        self.percDown(1)
        return retval
```

```
def percDown(self,currentIndex):
    while True:
        if currentIndex*2 > self.currentSize:
            return
        minIndex = minChild(currentIndex)
        if self.heapList[currentIndex] >
                self.heapList[minIndex]:
            temp =
                 =
                 =
            currentIndex = minIndex
        else:
            return
```

f) What is the big-oh notation for delMin? $O(\log_2 n)$

Once we have a working `BinHeap`, then implementing the `PriorityQueue` class using a `BinHeap` is a piece of cake:

```
### File: priority_queue.py
from binheap import BinHeap

class PriorityQueue:
    def __init__(self):
        self._heap = BinHeap()

    def isEmpty(self):
        return self._heap.isEmpty()

    def enqueue(self, item):
        self._heap.insert(item)

    def dequeue(self):
        return self._heap.delMin()

    def size(self):
        return self._heap.size()

    def __str__(self):
        return str(self._heap)
```

```
>>> q = PriorityQueue()
>>> print(q)
[]
>>> q.enqueue(5)
>>> q.enqueue(1)
>>> q.enqueue(7)
>>> print(q)
[1, 5, 7]
>>> q.dequeue()
1
>>> print(q)
[5, 7]
```

3. A "list" is a generic term for a sequence of items in a linear arrangement. Unlike stacks, queues and deques access to list items is not limited to either end, but can be from any position in the list. The general terminology of a list is illustrated by:

"Abstract view of a list"

[0] [1] [2] [3] ← index/position in the list

'w' 'a' 'y' 'c'

↑ *head* ↑ *tail*

There are three broad categories of list operations that are possible:

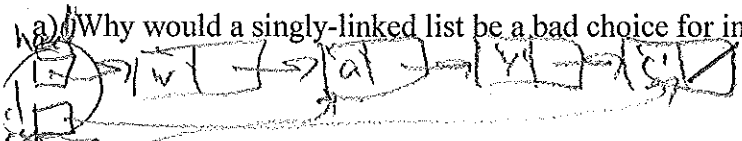
- **index-based operations** - the list is manipulated by specifying an index location, e.g.,
  myList.insert(3, item) # insert item at index 3 in myList
- **content-based operations** - the list is manipulated by specifying some content (i.e., item value), e.g.,
  myList.remove(item) # removes the item from the list based on its value
- **cursor-base operations** - a *cursor* (current position) can be moved around the list, and it is used to identify list items to be manipulated, e.g.,
  myList.first() # sets the cursor to the head item of the list
  myList.next() # moves the cursor one position toward the tail of the list
  myList.remove() # deletes the second item in the list because that's where the cursor is currently located

The following table summarizes the operations from the three basic categories on a list, L:

Index-based operations	**Content-based operations**	**cursor-based operations**
`L.insert(index, item)`	`L.add(item)`	`L.hasNext()`
`item = L[index]`	`L.remove(item)`	`L.next()`
`L[index] = newValue`	`L.search(item) #return Boolean`	`L.hasPrevious()`
`L.pop(index)`	`i = L.index(item)`	`L.previous()`
		`L.first()`
		`L.last()`
		`L.insert(item)`
		`L.replace(item)`
		`L.remove()`

Built-in Python lists are unordered with a mixture of index-based and content-based operations. We know they are implemented using a contiguous block of memory (i.e., an array). The textbook talks about an unordered list ADT, and a sorted list ADT which is more content-based. Both are implemented using a singly-linked list.

a) Why would a singly-linked list be a bad choice for implementing a cursor-based list ADT?



```
# Bradley N. Miller, David L. Ranum
# Introduction to Data Structures and Algorithms in Python
# Copyright 2005
#
import unittest

# this heap takes key value pairs, we will assume that the keys are integers
class BinHeap:
    def __init__(self):
        self.heapList = [0]
        self.currentSize = 0


    def buildHeap(self,alist):
        i = len(alist) // 2
        self.currentSize = len(alist)
        self.heapList = [0] + alist[:]
        print(len(self.heapList), i)
        while (i > 0):
            print(self.heapList, i)
            self.percDown(i)
            i = i - 1
        print(self.heapList,i)

    def percDown(self,i):
        while (i * 2) <= self.currentSize:
            mc = self.minChild(i)
            if self.heapList[i] > self.heapList[mc]:
                tmp = self.heapList[i]
                self.heapList[i] = self.heapList[mc]
                self.heapList[mc] = tmp
            i = mc

    def minChild(self,i):
        if i * 2 + 1 > self.currentSize:
            return i * 2
        else:
            if self.heapList[i * 2] < self.heapList[i * 2 + 1]:
                return i * 2
            else:
                return i * 2 + 1

    def percUp(self,i):
        while i // 2 > 0:
            if self.heapList[i] < self.heapList[i//2]:
               tmp = self.heapList[i // 2]
               self.heapList[i // 2] = self.heapList[i]
               self.heapList[i] = tmp
            i = i // 2

    def insert(self,k):
```