Data Structures - Test 2

Question 1. (10 points) What is printed by the following program?

```python
def recFn(a, b):
    print(a, b)
    if a > b:
        return a
    elif a == b:
        return a + b
    else:
        return recFn(a + 1, b - 1)

print("Result = ", recFn(1, 7))
```

Output:

```
1 7
2 6
3 5
4 4
Result = 14
```

Question 2. (8 points) Write a recursive Python function to compute the following mathematical function, G(n):

- \( G(n) = 1 \) for all values of \( n \leq 0 \)
- \( G(n) = 2 \) for \( n = 1 \)
- \( G(n) = G(n-5) + G(n-4) + G(n-3) \) for all values of \( n > 1 \).

```python
def G(n):
    if n <= 0:
        return 1
    elif n == 1:
        return 2
    else:
        return G(n-5) + G(n-4) + G(n-3)
```

Question 3. (7 points)

a) For the above recursive function \( G(n) \), complete the calling-tree for \( G(7) \).

b) What is the value of \( G(7) \)?

\( 14 \)

c) What is the maximum height of the run-time stack when calculating \( G(7) \) recursively?

\( 3 \)
Fall 2014

Question 4. (10 points.) Consider the following timings of the recursive $g(n)$ function from question 2.

<table>
<thead>
<tr>
<th>Value of $n$</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to calculate $g(n)$ in seconds</td>
<td>0.05</td>
<td>0.56</td>
<td>8.78</td>
<td>145.98</td>
<td>2,430.46</td>
</tr>
</tbody>
</table>

a) Explain why the recursive $g(n)$ function from question 2 is so slow.

It solves the same smaller problems many times.

b) Explain how we could speed up the calculation of $g(n)$? (no code is necessary just an explanation)

Dynamic programming -- solve the smaller problems only once and store their answers in a list so their answers can be looked up if needed for larger problems. Start with list of use 1 2 0 to get 1 2 1 2

calculate from 2 up to $n$.

Question 5. (15 points) Consider the timings (in seconds) of various ascending, bubble sorting algorithms on 10,000 items.

<table>
<thead>
<tr>
<th>Type of bubble sorting algorithm</th>
<th>Initial Ordering of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Descending</td>
</tr>
<tr>
<td>bubble - no check to stop early</td>
<td>23.2</td>
</tr>
<tr>
<td>bubble - with a check to stop early</td>
<td>24.1</td>
</tr>
</tbody>
</table>

a) Why does the bubble sort with no check to stop early take less time on the ascending ordered list than it does on the descending ordered list?

As unsort part is scanned, no swap are needed since the items are already in ascending order.

b) Why does the bubble sort with a check to stop early take A LOT less time on the ascending ordered list than the descending ordered list?

After one scan down the unsorted array without any swaps, the sort quits.

c) Why does the bubble sort with no check to stop early take less time on the descending ordered list than the bubble sort with a check to stop early on the descending ordered list?

Bubble sort with no check does not execute the setting of the flag and checking the flag.
Question 6. (20 points) In class we discussed the following insertion sort code which sorts in ascending order (smallest to largest) and builds the sorted part on the left-hand side of the list.

```python
def insertionSort(myList):
    for firstUnsortedIndex in range(1, len(myList)):
        itemToInsert = myList[firstUnsortedIndex]
        testIndex = firstUnsortedIndex - 1

        while testIndex >= 0 and myList[testIndex] > itemToInsert:
            myList[testIndex+1] = myList[testIndex]
            testIndex = testIndex - 1

        myList[testIndex + 1] = itemToInsert
```

For this question write a variation of the above insert sort that:
- sorts in **descending order** (largest to smallest)
- builds the **sorted part on the right-hand side** of the list, i.e.,

<table>
<thead>
<tr>
<th>Unsorted Part</th>
<th>Item To Insert</th>
<th>Sorted Part</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>40</td>
<td>0 10</td>
</tr>
</tbody>
</table>

```python
def insertionSort(myList):
    for lastUnsorted in range(len(myList)-2, -1, -1):
        itemToInsert = myList[lastUnsorted]
        testIndex = lastUnsorted + 1

        while testIndex < len(myList) and myList[testIndex] > itemToInsert:
            myList[testIndex-1] = myList[testIndex]
            testIndex = testIndex + 1

        myList[testIndex] = itemToInsert
```
Question 7. (15 points) Recall the common rehashing strategies we discussed for open-address hashing:

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>quadratic</td>
<td>probing</td>
</tr>
<tr>
<td>home address +</td>
<td>(rehash attempt #)^2 + (rehash attempt #) / 2] % (hash table size), where the hash table size is a power of 2. Integer division is used above</td>
</tr>
</tbody>
</table>

a) Insert “Paul Gray” and then “Sarah Diesburg” using Linear (on left) and Quadratic (on right) probing.

b) What is the purpose of requiring a hash table size that is a power of 2 when using quadratic probing?

So the whole table, i.e., all of the indexes, are probed before any are repeated.

Question 8. (15 points) Use the below diagram to explain the worst-case big-oh notation of merge sort. Assume “n” items to sort.

Overall \( O(n \log_2 n) \)