1. The textbook solves the coin-change problem with the following code (note the "set-builder-like" notation): {c | c ∈ coinValueList and c ≤ change}

```
def recMC(change, coinValueList):
    global backtrackingNodes
    backtrackingNodes += 1
    minCoins = change
    if change in coinValueList:
        return 1
    else:
        for i in [c for c in coinValueList if c <= change]:
            numCoins = 1 + recMC(change - i, coinValueList)
            if numCoins < minCoins:
                minCoins = numCoins
    return minCoins
```

Results of running this code:

```
Change Amount: 63 Coin types: [1, 5, 10, 25]
Run-time: 70.689 seconds
Fewest number of coins 6
Number of Backtracking Nodes: 67,716,925
```

I removed the fancy set-builder notation and replaced it with a simple `if`-statement check:

```
def recMC(change, coinValueList):
    global backtrackingNodes
    backtrackingNodes += 1
    minCoins = change
    if change in coinValueList:
        return 1
    else:
        for i in coinValueList:
            if i <= change:
                numCoins = 1 + recMC(change - i, coinValueList)
                if numCoins < minCoins:
                    minCoins = numCoins
    return minCoins
```

Results of running this code:

```
Change Amount: 63 Coin types: [1, 5, 10, 25]
Run-time: 45.815 seconds
Fewest number of coins 6
Number of Backtracking Nodes: 67,716,925
```

a) Why is the second version so much "faster"?

It doesn't need to build a list of coin values that are <= change for each call to recMC.

b) Why does it still take a long time?

still does 67,716,925 recursive calls to recMC

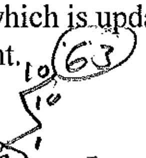
2. To speed the recursive backtracking algorithm, we can prune unpromising branches. The general recursive backtracking algorithm for optimization problems (e.g., fewest number of coins) looks something like:

```
Backtrack( recursionTreeNode p ) {
  for each child c of p do                          # each c represents a possible choice
      if promising(c) then                          # c is "promising" if it could lead to a better solution
          if c is a solution that's better than best then   # check if this is the best solution found so far
              best = c                              # remember the best solution
          else
              Backtrack(c)                          # follow a branch down the tree
          end if
      end if
  end for
} // end Backtrack
```

General Notes about Backtracking:

- The depth-first nature of backtracking only stores information about the current branch being explored on the run-time stack, so the memory usage is "low" eventhough the # of recursion tree nodes might be exponential (2^n).
- Each node of the search-space (recursive-call) tree maintains the state of a partial solution. In general the partial solution state consists of potentially large arrays that change little between parent and child. To avoid having multiple copies of these arrays, a reference to a single "global" array can be maintained which is updated before we go down to the child (via a recursive call) and undone when we backtrack to the parent.

a) For the coin-change problem, what defines the current state of a search-space tree node?



current change amount, number coins already returned (3) and which coins (10,10,1)

b) When would a "child" tree node NOT be promising? If we already have a solution, say 5 coins solution, and we have already given back 4 coins and have a positive change amount, then we cannot hope to do better than our previously found 5 coin solution.

3. Consider the output of running the backtracking code with pruning (next page) twice with a change amount of 63 cents.

Change Amount: 63 Coin types: [1, 5, 10, 25] Run-time: 0.036 seconds Fewest number of coins 6 The number of each type of coins is: number of 1-cent coins is 3 number of 5-cent coins is 0 number of 10-cent coins is 1 number of 25-cent coins is 2 Number of Backtracking Nodes: 4831	Change Amount: 63 Coin types: [25, 10, 5, 1] Run-time: 0.003 seconds Fewest number of coins 6 The number of each type of coins is: number of 25-cent coins is 2 number of 10-cent coins is 1 number of 5-cent coins is 0 number of 1-cent coins is 3 Number of Backtracking Nodes: 310

a) Explain why ordering the coins from largest to smallest produced faster results.

The [1,5,10,25] version's first solution found will be 63 pennies. which is not too helpful for pruning. The [25,10,5,1] version's first solution found will be our greedy solution (25,25,10,1,1,1) six-coin solution which is best.

b) For coins of [50, 25, 12, 10, 5, 1] typical timings:

Change Amount	Run-Time (seconds)	Number of Tree Nodes
399	8.88	2,015,539
409	55.17	12,093,221
419	318.56	72,558,646

Why the exponential growth in run-time?

4. As with Fibonacci, the coin-change problem can benefit from dynamic program since it was slow due to solving the same problems over-and-over again. Recall the general idea of dynamic programming:

- Solve smaller problems before larger ones
- store their answers
- look-up answers to smaller problems when solving larger subproblems, so each problem is solved only once

a) To solve the coin-change problem using dynamic programming, we need to answer the questions:

- What is the smallest problem? 0 change amount 29¢ {1,5,10,12,25,50}
- Where do we store the answers to the smaller problems? list

```python
backtrackingNodes = 0  # profiling variable to track number of state-space tree nodes

def solveCoinChange(changeAmt, coinTypes):

    def backtrack(changeAmt, numberOfEachCoinType, numberOfCoinsSoFar, solutionFound, bestFewestCoins, bestNumberOfEachCoinType):
        global backtrackingNodes
        backtrackingNodes += 1

        for index in range(len(coinTypes)):
            smallerChangeAmt = changeAmt - coinTypes[index]
            if promising(smallerChangeAmt, numberOfCoinsSoFar+1, solutionFound, bestFewestCoins):
                if smallerChangeAmt == 0:  # a solution is found
                    if (not solutionFound) or numberOfCoinsSoFar + 1 < bestFewestCoins: # check if its best
                        bestFewestCoins = numberOfCoinsSoFar+1
                        bestNumberOfEachCoinType = [] + numberOfEachCoinType
                        bestNumberOfEachCoinType[index] += 1
                        solutionFound = True
                else:
                    # call child with updated state information
                    smallerChangeAmtNumberOfEachCoinType = [] + numberOfEachCoinType
                    smallerChangeAmtNumberOfEachCoinType[index] += 1

                    solutionFound, bestFewestCoins, bestNumberOfEachCoinType = backtrack(smallerChangeAmt, smallerChangeAmtNumberOfEachCoinType,
                                                                                         numberOfCoinsSoFar + 1, solutionFound, bestFewestCoins,
                                                                                         bestNumberOfEachCoinType)
        return solutionFound, bestFewestCoins, bestNumberOfEachCoinType
    # end def backtrack

    def promising(changeAmt, numberOfCoinsReturned, solutionFound, bestFewestCoins):
        if changeAmt < 0:
            return False
        elif changeAmt == 0:
            return True
        else:  # changeAmt > 0
            if solutionFound and numberOfCoinsReturned+1 >= bestFewestCoins:
                return False
            else:
                return True

    # Body of solveCoinChange
    numberOfEachCoinType = []        # set-up initial "current state" information
    numberOfCoinsSoFar = 0
    solutionFound = False
    bestFewestCoins = -1
    bestNumberOfEachCoinType = None


    numberOfEachCoinType = []
    for coin in coinTypes:
        numberOfEachCoinType.append(0)
    numberOfCoinsSoFar = 0
    solutionFound = False
    bestFewestCoins = -1
    bestNumberOfEachCoinType = None


    solutionFound, bestFewestCoins, bestNumberOfEachCoinType = backtrack(changeAmt, numberOfEachCoinType, numberOfCoinsSoFar, solutionFound,
                                                                         bestFewestCoins, bestNumberOfEachCoinType)
    return bestFewestCoins, bestNumberOfEachCoinType
```

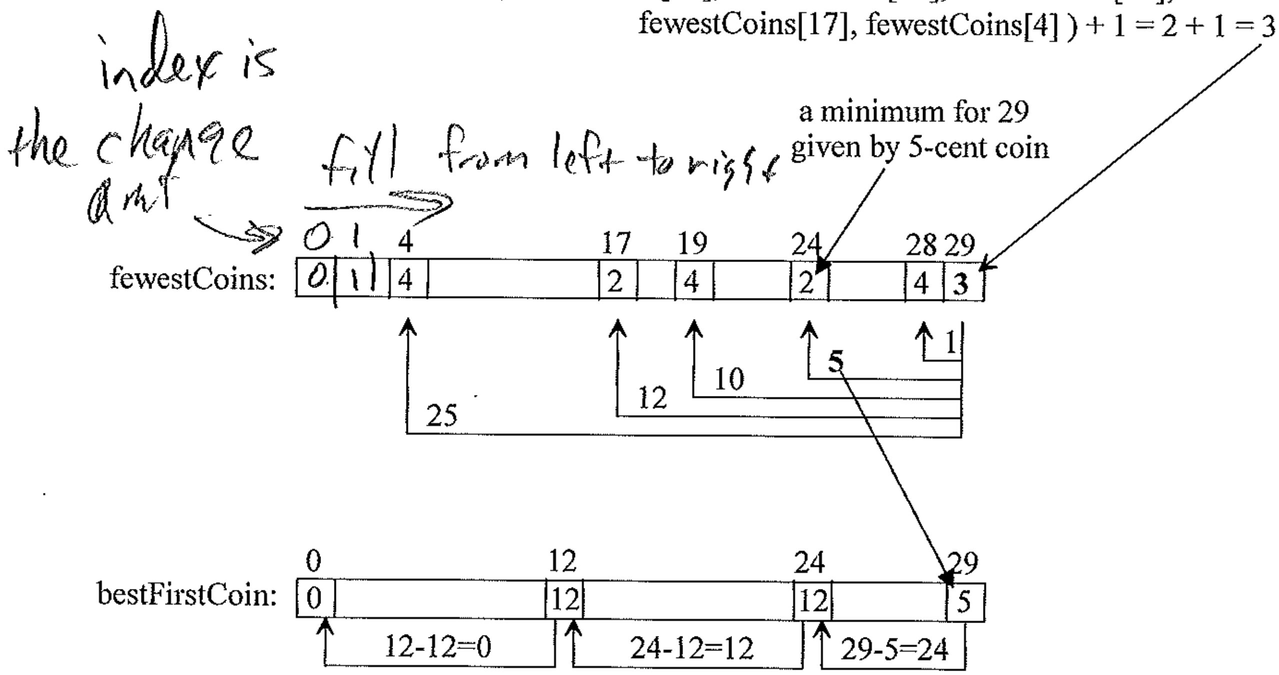
Dynamic Programming Coin-change Algorithm:

I. Fills an array fewestCoins from 0 to the amount of change. An element of fewestCoins stores the fewest number of coins necessary for the amount of change corresponding to its index value.

For 29-cents using the set of coin types {1, 5, 10, 12, 25, 50}, the dynamic programming algorithm would have previously calculated the fewestCoins for the change amounts of 0, 1, 2, ..., up to 28 cents.

II. If we record the best, first coin to return for each change amount (found in the "minimum" calculation) in an array bestFirstCoin, then we can easily recover the actual coin types to return.

fewestCoins[29] = minimum(fewestCoins[28], fewestCoins[24], fewestCoins[19], fewestCoins[17], fewestCoins[4]) + 1 = 2 + 1 = 3



Extract the coins in the solution for 29-cents from bestFirstCoin[29], bestFirstCoin[24], and bestFirstCoin[12]

b) Extend the lists through 32-cents.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
fewestCoins:	0	1	2	3	4	1	2	3	4	5	1	2	1	2	3	2	3	2	3	4	2	3	2	3	2	1	2	3	4	3	2	3	3

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
bestFirstCoin:	0	1	1	1	1	5	1	1	1	1	10	1	12	1	1	5	1	5	1	1	10	1	10	1	12	25	1	1	1	5	5	1	10

c) What coins are in the solution for 32-cents? 10, 10, 12

1. Consider the following sequential search (linear search) code:

Textbook's Listing 5.1	Faster sequential search code

`def sequentialSearch(alist, item):`
`    """ Sequential search of unorder list """`
`    pos = 0`
`    found = False`
 	
`    while pos < len(alist) and not found:`
`        if alist[pos] == item:`
`            found = True`
`        else:`
`            pos = pos+1`
 	
`    return found`
	`def linearSearch(aList, target):`
	`    """Returns the index of target in aList`
	`       or -1 if target is not in aList"""`
	`    for position in range(len(aList)):`
	`        if target == aList[position]:`
	`            return position`
	`    return -1`

a) What is the *basic operation* of a search? comparison of target to list item

b) For the following aList value, which target value causes linearSearch to loop the fewest ("best case") number of times? 10

best-case O(1)

	0	1	2	3	4	5	6	7	8	9	10
aList:	10	15	28	42	60	69	75	88	90	93	97

B(1)

c) For the above aList value, which target value causes linearSearch to loop the most ("worst case") number of times?

O(n) 97

where n ≡ # items in List or unsuccessful search

d) For a *successful search* (i.e., target value in aList), what is the "average" number of loops?

$O(\frac{n}{2}) \equiv O(n)$

Textbook's Listing 5.2	Faster sequential search code

`def orderedSequentialSearch(alist, item):`
`    """ Sequential search of order list """`
`    pos = 0`
`    found = False`
`    stop = False`
`    while pos < len(alist) and not found and not stop:`
`        if alist[pos] == item:`
`            found = True`
`        else:`
`            if alist[pos] > item:`
`                stop = True`
`            else:`
`                pos = pos+1`
 	
`    return found`
	`def linearSearchOfSortedList(target, aList):`
	`    """Returns the index position of target in`
	`    sorted aList or -1 if target is not in aList"""`
	`    breakOut = False`
	`    for position in range(len(aList)):`
	`        if target <= aList[position]:`
	`            breakOut = True`
	`            break`
	
	`    if not breakOut:`
	`        return -1`
	`    elif target == aList[position]:`
	`        return position`
	`    else:`
	`        return -1`

e) The above version of linear search assumes that aList is sorted in ascending order. When would this version perform better than the original linearSearch at the top of the page?

We can stop early on some unsuccessful searches when alist[pos] > target item.

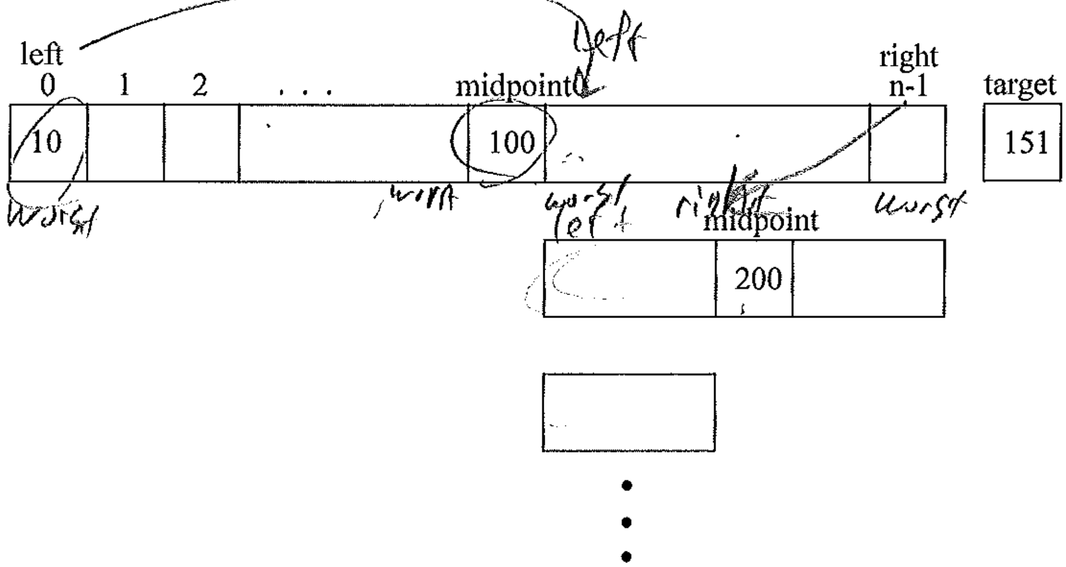
2. Consider the following binary search code:

Textbook's Listing 5.3	Faster binary search code

<pre>def binarySearch(alist, item):
    first = 0
    last = len(alist)-1
    found = False
 	
    while first<=last and not found:
        midpoint = (first + last)//2
        if alist[midpoint] == item:
            found = True
        else:
            if item < alist[midpoint]:
                last = midpoint-1
            else:
                first = midpoint+1
 	
    return found</pre>
	<pre>def binarySearch(target, lyst):
	    """Returns the position of the target
	      item if found, or -1 otherwise."""
	    left = 0
	    right = len(lyst) - 1
	    while left <= right:
	        midpoint = (left + right) // 2
	        if target == lyst[midpoint]:
	            return midpoint
	        elif target < lyst[midpoint]:
	            right = midpoint - 1
	        else:
	            left = midpoint + 1
	    return -1</pre>

a) "Trace" binary search to determine the worst-case basic total number of comparisons?

loop #	worst-case # elements remaining
1	"n"
2	n/2
3	n/4
	n/8
	⋮
	1



b) What is the worst-case big-oh for binary search? $O(\log_2 n)$

c) What is the best-case big-oh for binary search? $O(1)$

d) What is the average-case (expected) big-oh for binary search? $O(\log_2 n)$

e) If the list size is 1,000,000, then what is the maximun number of comparisons of list items on a *successful search*?

$2^{20} = 1,000,000$

$\log_2 1,000,000 = 20$

f) If the list size is 1,000,000, then how many comparisons would you expect on an *unsuccessful search*?

$= 20$