1. The textbook solves the coin-change problem with the following code (note the "set-builder-like" notation):

$\{c \mid c \in \text{coinValueList and } c \leq \text{change}\}$

```
def recMC(change, coinValueList):
    global backtrackingNodes
    backtrackingNodes += 1
    minCoins = change
    if change in coinValueList:
        return 1
    else:
        for i in [c for c in coinValueList if c <= change]:
            numCoins = 1 + recMC(change - i, coinValueList)
            if numCoins < minCoins:
                minCoins = numCoins
    return minCoins
```

Results of running this code:

```
Change Amount: 63 Coin types: [1, 5, 10, 25]
Run-time: 70.689 seconds
Fewest number of coins 6
Number of Backtracking Nodes: 67,716,925
```

I removed the fancy set-builder notation and replaced it with a simple `if`-statement check:

```
def recMC(change, coinValueList):
    global backtrackingNodes
    backtrackingNodes += 1
    minCoins = change
    if change in coinValueList:
       return 1
    else:
       for i in coinValueList:
           if i <= change:
               numCoins = 1 + recMC(change - i, coinValueList)
               if numCoins < minCoins:
                   minCoins = numCoins
    return minCoins
```

Results of running this code:

```
Change Amount: 63 Coin types: [1, 5, 10, 25]
Run-time: 45.815 seconds
Fewest number of coins 6
Number of Backtracking Nodes: 67,716,925
```

a) Why is the second version so much "faster"?

b) Why does it still take a long time?

2. To speed the recursive backtracking algorithm, we can prune unpromising branches. The general recursive backtracking algorithm for optimization problems (e.g., fewest number of coins) looks something like:

```
Backtrack( recursionTreeNode p ) {
   for each child c of p do                              # each c represents a possible choice
       if promising(c) then                              # c is "promising" if it could lead to a better solution
           if c is a solution that's better than best then   # check if this is the best solution found so far
               best = c                                  # remember the best solution
           else
               Backtrack(c)                              # follow a branch down the tree
           end if
       end if
   end for
} // end Backtrack
```

General Notes about Backtracking:

- The depth-first nature of backtracking only stores information about the current branch being explored on the run-time stack, so the memory usage is "low" eventhough the # of recursion tree nodes might be exponential (2^n).
- Each node of the search-space (recursive-call) tree maintains the state of a partial solution. In general the partial solution state consists of potentially large arrays that change little between parent and child. To avoid having multiple copies of these arrays, a reference to a single "global" array can be maintained which is updated before we go down to the child (via a recursive call) and undone when we backtrack to the parent.

a) For the coin-change problem, what defines the current state of a search-space tree node?

b) When would a “child” tree node NOT be promising?

3. Consider the output of running the backtracking code with pruning (next page) twice with a change amount of 63 cents.

```
Change Amount: 63  Coin types: [1, 5, 10, 25]
Run-time: 0.036 seconds
Fewest number of coins 6
The number of each type of coins is:
number of 1-cent coins is 3
number of 5-cent coins is 0
number of 10-cent coins is 1
number of 25-cent coins is 2
Number of Backtracking Nodes: 4831
```

```
Change Amount: 63  Coin types: [25, 10, 5, 1]
Run-time: 0.003 seconds
Fewest number of coins 6
The number of each type of coins is:
number of 25-cent coins is 2
number of 10-cent coins is 1
number of 5-cent coins is 0
number of 1-cent coins is 3
Number of Backtracking Nodes: 310
```

a) Explain why ordering the coins from largest to smallest produced faster results.

b) For coins of [50, 25, 12, 10, 5, 1] typical timings:

Change Amount	Run-Time (seconds)	Number of Tree Nodes
399	8.88	2,015,539
409	55.17	12,093,221
419	318.56	72,558,646

Why the exponential growth in run-time?

4. As with Fibonacci, the coin-change problem can benefit from dynamic program since it was slow due to solving the same problems over-and-over again. Recall the general idea of dynamic programming:

- Solve smaller problems before larger ones
- store their answers
- look-up answers to smaller problems when solving larger subproblems, so each problem is solved only once

a) To solve the coin-change problem using dynamic programming, we need to answer the questions:

- What is the smallest problem?

- Where do we store the answers to the smaller problems?

```
backtrackingNodes = 0  # profiling variable to track number of state-space tree nodes

def solveCoinChange(changeAmt, coinTypes):

    def backtrack(changeAmt, numberOfEachCoinType, numberOfCoinsSoFar, solutionFound, bestFewestCoins, bestNumberOfEachCoinType):
        global backtrackingNodes
        backtrackingNodes += 1

        for index in range(len(coinTypes)):
            smallerChangeAmt = changeAmt - coinTypes[index]
            if promising(smallerChangeAmt, numberOfCoinsSoFar+1, solutionFound, bestFewestCoins):
                if smallerChangeAmt == 0:  # a solution is found
                    if (not solutionFound) or numberOfCoinsSoFar + 1 < bestFewestCoins: # check if its best
                        bestFewestCoins = numberOfCoinsSoFar+1
                        bestNumberOfEachCoinType = [] + numberOfEachCoinType
                        bestNumberOfEachCoinType[index] += 1
                        solutionFound = True
                else:
                    # call child with updated state information
                    smallerChangeAmtNumberOfEachCoinType = [] + numberOfEachCoinType
                    smallerChangeAmtNumberOfEachCoinType[index] += 1

                    solutionFound, bestFewestCoins, bestNumberOfEachCoinType = backtrack(smallerChangeAmt, smallerChangeAmtNumberOfEachCoinType,
                                                                                         numberOfCoinsSoFar + 1, solutionFound, bestFewestCoins,
                                                                                         bestNumberOfEachCoinType)
        return solutionFound, bestFewestCoins, bestNumberOfEachCoinType
    # end def backtrack

    def promising(changeAmt, numberOfCoinsReturned, solutionFound, bestFewestCoins):
        if changeAmt < 0:
            return False
        elif changeAmt == 0:
            return True
        else:  # changeAmt > 0
            if solutionFound and numberOfCoinsReturned+1 >= bestFewestCoins:
                return False
            else:
                return True

    # Body of solveCoinChange
    numberOfEachCoinType = []        # set-up initial "current state" information
    numberOfCoinsSoFar = 0
    solutionFound = False
    bestFewestCoins = -1
    bestNumberOfEachCoinType = None


    numberOfEachCoinType = []
    for coin in coinTypes:
        numberOfEachCoinType.append(0)
    numberOfCoinsSoFar = 0
    solutionFound = False
    bestFewestCoins = -1
    bestNumberOfEachCoinType = None


    solutionFound, bestFewestCoins, bestNumberOfEachCoinType = backtrack(changeAmt, numberOfEachCoinType, numberOfCoinsSoFar, solutionFound,
                                                                         bestFewestCoins, bestNumberOfEachCoinType)
    return bestFewestCoins, bestNumberOfEachCoinType
```

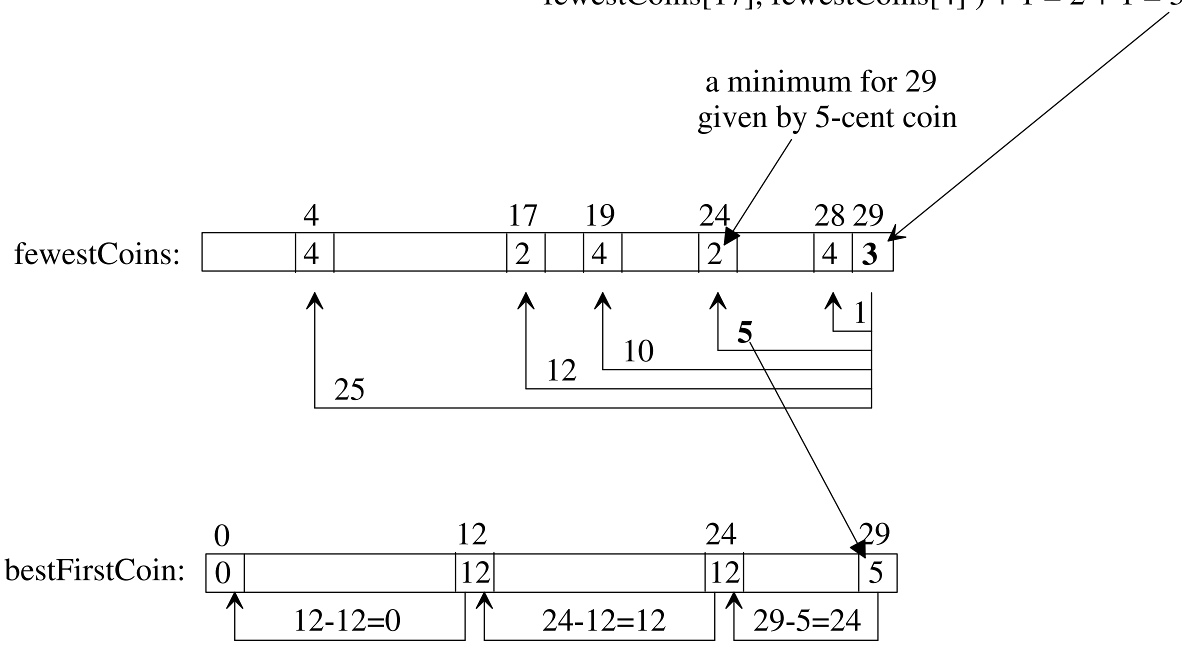
Dynamic Programming Coin-change Algorithm:

I. Fills an array fewestCoins from 0 to the amount of change. An element of fewestCoins stores the fewest number of coins necessary for the amount of change corresponding to its index value.

For 29-cents using the set of coin types {1, 5, 10, 12, 25, 50}, the dynamic programming algorithm would have previously calculated the fewestCoins for the change amounts of 0, 1, 2, ..., up to 28 cents.

II. If we record the best, first coin to return for each change amount (found in the "minimum" calculation) in an array bestFirstCoin, then we can easily recover the actual coin types to return.

fewestCoins[29] = minimum(fewestCoins[28], fewestCoins[24], fewestCoins[19], fewestCoins[17], fewestCoins[4]) + 1 = 2 + 1 = 3



Extract the coins in the solution for 29-cents from bestFirstCoin[29], bestFirstCoin[24], and bestFirstCoin[12]

b) Extend the lists through 32-cents.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
fewestCoins:	0	1	2	3	4	1	2	3	4	5	1	2	1	2	3	2	3	2	3	4	2	3	2	3	2	1	2	3	4	3			

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
bestFirstCoin:	0	1	1	1	1	5	1	1	1	1	10	1	12	1	1	5	1	5	1	1	10	1	10	1	12	25	1	1	1	5			

c) What coins are in the solution for 32-cents?