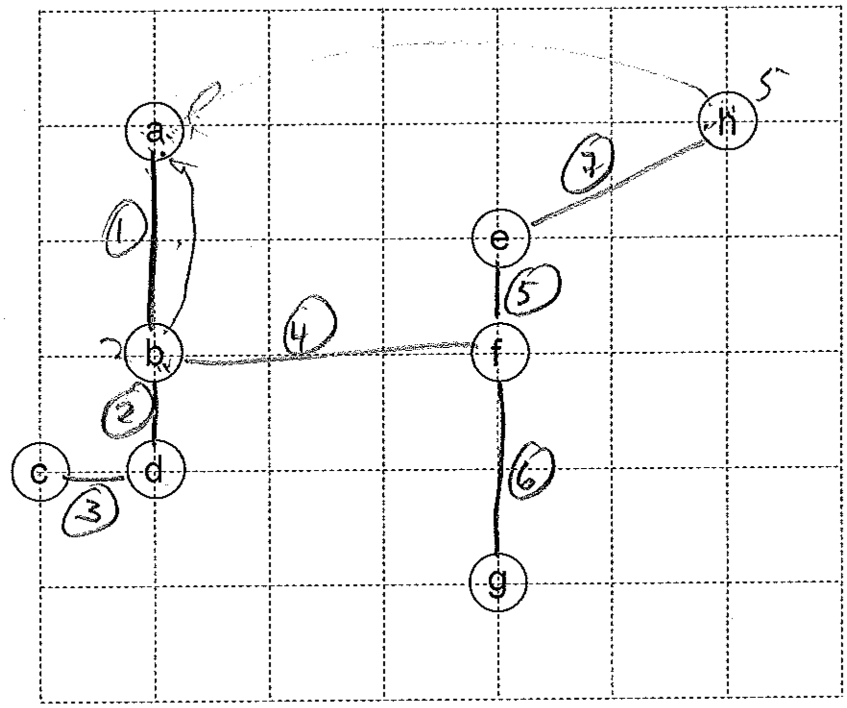
1. Suppose you had a map of settlements on the planet X
(Assume edges could connecting all vertices with their Euclidean distances as their costs)



We want to build roads that allow us to travel between any pair of cities. Because resources are scarce, we want the total length of all roads build to be minimal. Since all cities will be connected anyway, it does not matter where we start, but assume we start at "a".

a) Assuming we start at city "a" which city would you connect first? b Why this city?

It is the closest city to 'a'

b) What city would you connect next to expand your partial road network?

next closest to anything in partial road system (priority queue / min. heap)

c) What would be some characteristics of the resulting "graph" after all the cities are connected?

"connected" graph with no cycles ⇒ "spanning" tree

d) Does your algorithm come up with the overall best (globally optimal) result?

bestest / min. spanning tree - MST

2. Prim's algorithm for determining the minimum-spanning tree (MST) of a graph is another example of a *greedy algorithm*. Unlike divide-and-conquer and dynamic programming algorithms, greedy algorithms DO NOT divide a problem into smaller subproblems. Instead a greedy algorithm builds a solution by making a sequence of choices that look best ("locally" optimal) at the moment without regard for past or future choices (no backtracking to fix bad choices).

a) What greedy criteria does Prim's algorithm use to select the next vertex and edge to the partial minimum spanning tree? *vertex closest to any vertex in partial MST*

b) Consider the textbook's Prim's Algorithm code (Listing 7.12 p. 346) which is incorrect.

```
def prim(G,start):
    pq = PriorityQueue()
    for v in G:
        v.setDistance(sys.maxsize)
        v.setPred(None)
    start.setDistance(0)
    pq.buildHeap([(v.getDistance(),v) for v in G])
    while not pq.isEmpty():
        currentVert = pq.delMin()
        for nextVert in currentVert.getConnections():
            newCost = currentVert.getWeight(nextVert) \
                    + currentVert.getDistance()
            if v in pq and newCost<nextVert.getDistance():
                nextVert.setPred(currentVert)
                nextVert.setDistance(newCost)
                pq.decreaseKey(nextVert,newCost)
```

(handwritten corrections in the code: (1) PriorityQueue() crossed out → BinHeap(); (4) PriorityQueueEntry(v.getDistance(), v) in buildHeap; delMin().getValue(); "+ currentVert.getDistance()" crossed out; v → PriorityQueueEntry(newCost, nextVert) / (5) __contains__; pq.decreaseKey(PriorityQueueEntry(newCost, nextVert)) with "distance")

c) What is wrong with the code? (Fix the above code.)

(1) BinHeap instead of PriorityQueue
(2) newCost calculation wrong
(3) "v" should nextVert in if-statement
(4) pq should a heap of PriorityQueueEntry objects
(5) add __contains__ to BinHeap + decrease key

3. To avoid "massive" changes to the BinHeap class, it can store `PriorityQueueEntry` objects:

```
class PriorityQueueEntry:
    def __init__(self,x,y):
        self.key = x      distance
        self.val = y      vertex

    def getKey(self):
        return self.key

    def getValue(self):
        return self.val

    def setValue(self, newValue):
        self.val = newValue
```

```
    def __lt__(self,other):
        return self.key < other.key

    def __gt__(self,other):
        return self.key > other.key

    def __eq__(self, other):
        return self.val == other.val

    def __hash__(self):
        return self.key
```

a) Update the above Prim's algorithm code to use `PriorityQueueEntry` objects.

b) Why do the `__lt__` and `__gt__` methods compare `key` attributes, but `__eq__` compare `val` attributes?

distance use "<" in percUp; search __contains__ by vertex/val part

c) When used for Prim's algorithm what type of objects are the `vals` compared by `__eq__`? Vertex objects

d) What changes to the Graph and Vertex classes need to be made? none ?

(, 'h')

e) Complete the `__contains__` and `decreaseKey` methods.

```
class BinHeap:
    def __init__(self):
        self.heapList = [0]
        self.currentSize = 0
        self._foundIndex = 0
    def buildHeap(self,alist):
        i = len(alist) // 2
        self.currentSize = len(alist)
        self.heapList = [0] + alist[:]
        while (i > 0):
            self.percDown(i)
            i = i - 1

    def percDown(self,i):
        while (i * 2) <= self.currentSize:
            mc = self.minChild(i)
            if self.heapList[i] > self.heapList[mc]:
                tmp = self.heapList[i]
                self.heapList[i] = self.heapList[mc]
                self.heapList[mc] = tmp
            i = mc

    def minChild(self,i):
        if i * 2 + 1 > self.currentSize:
            return i * 2
        else:
            if self.heapList[i*2] < self.heapList[i*2+1]:
                return i * 2
            else:
                return i * 2 + 1

    def percUp(self,i):
        while i // 2 > 0:
            if self.heapList[i] < self.heapList[i//2]:
               tmp = self.heapList[i // 2]
               self.heapList[i // 2] = self.heapList[i]
               self.heapList[i] = tmp
            i = i // 2

    def insert(self,k):
        self.heapList.append(k)
        self.currentSize = self.currentSize + 1
        self.percUp(self.currentSize)

    def delMin(self):
        retval = self.heapList[1]
        self.heapList[1] = self.heapList[self.currentSize]
        self.currentSize = self.currentSize - 1
        self.heapList.pop()
        self.percDown(1)
        return retval

    def isEmpty(self):
        return self.currentSize == 0

    def size(self):
        return self.currentSize

    def __str__(self):
        return str(self.heapList[1:])
```

```
def __contains__(self, value):      # value: priorityQueueEntry
    for index in range(1, len(self.heapList)):
        if value == self.heapList[index]:
            self._foundIndex = index
            return True
    return False

def decreaseKey(self, decreasedValue):
      """Precondition:  decreasedValue
         in heap already"""

    if decreasedValue not in self:
        raise ValueError("                ")
    self.heapList[self._foundIndex] = decreasedValue
    self.percUp(self._foundIndex)
```