A Deque (pronounced “Deck”) is a linear data structure which behaves like a double-ended queue, i.e., it allows adding or removing items from either the front or the rear of the Deque.

1. One possible implementation of a Deque would be to use a Python list to store the Deque items such that
   - the rear item is **always stored at index 0**,
   - the front item is always stored at the highest index (or -1)

```
items: [ ]
0 1 2 3
(a)  'b' 'c' 'd'

```

a) Complete the `__init__` method and determine the big-oh, $O()$, for each Deque operation, assuming the above implementation. Let $n$ be the number of items in the Deque.

<table>
<thead>
<tr>
<th>isEmpty</th>
<th>addFront</th>
<th>removeFront</th>
<th>addRear</th>
<th>removeRear</th>
<th>size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Write the methods for the `addRear` and `removeRear` operation.

```python
def addRear(self, newItem):
    pass
def removeRear(self):
    pass
```

2. An alternative implementation of a Deque would be a linked implementation as in:

```
LinkedDeque Object           data   next   data   next   data   next   data   next
                                 'a'       'b'       'c'       'd'
_rear:                         ____________ ____________ ____________ ____________
__front:                       ____________ ____________ ____________ ____________ 4
__size:                        4
```

c) Complete the `__init__` method and determine the big-oh, $O()$, for each Deque operation assuming the above linked implementation. Let $n$ be the number of items in the Deque.

<table>
<thead>
<tr>
<th>isEmpty</th>
<th>addFront</th>
<th>removeFront</th>
<th>addRear</th>
<th>removeRear</th>
<th>size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Suggest an improvement to the above linked implementation of the Deque to speed up some of its operations.
from node import Node

class Node2Way(Node):
    def __init__(self, initdata):
        Node.__init__(self, initdata)
        self.previous = None

    def getPrevious(self):
        return self.previous

    def setPrevious(self, newprevious):
        self.previous = newprevious

3. An alternative implementation of a Deque would be a doubly-linked implementation as in:

DoublyLinkedListDeque Object

_a  rear: previous data next previous data next previous data next previous data next
_ front: 'a' 'b' 'c' 'd'
_size: 4

a) Determine the big-oh, $O()$, for each Deque operation assuming the above doubly-linked implementation. Let $n$ be the number of items in the Deque.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Big-Oh</th>
</tr>
</thead>
<tbody>
<tr>
<td>isEmpty</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>addFront</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>removeFront</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>addRear</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>removeRear</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>size</td>
<td>$O(1)$</td>
</tr>
</tbody>
</table>

4. A priority queue has the same operations as a regular queue, except the items are NOT returned in the FIFO (first-in, first-out) order. Instead, each item has a priority that determines the order they are removed. A hospital emergence room operates like a priority queue -- the person with the most serious injury has highest priority even if they just arrived.

a) Suppose that we have a priority queue with integer priorities such that the smallest integer corresponds to the highest priority. For the following priority queue, which item would be dequeued next?

priority queue:

40
10
79
30
13
5

b) To implement a priority queue, we could use an unordered Python list. If we did, what would be the big-oh notation for each of the following methods: (justify your answer)

- enqueue:
- dequeue:

c) To implement a priority queue, we could use a Python list order by priorities in descending order. If we did, what would be the big-oh notation for each of the following methods: (justify your answer)

- enqueue:
- dequeue