

1. Consider the partial `TreeNode` class and partial `BinarySearchTree` class.

```

class TreeNode:
    def __init__(self, key, val, left=None, right=None,
                 parent=None):

        self.key = key
        self.payload = val
        self.leftChild = left
        self.rightChild = right
        self.parent = parent

    def hasLeftChild(self):
        return self.leftChild

    def hasRightChild(self):
        return self.rightChild

    def isLeftChild(self):
        return self.parent and \
            self.parent.leftChild == self

    def isRightChild(self):
        return self.parent and \
            self.parent.rightChild == self

    def isRoot(self):
        return not self.parent

    def isLeaf(self):
        return not (self.rightChild or self.leftChild)

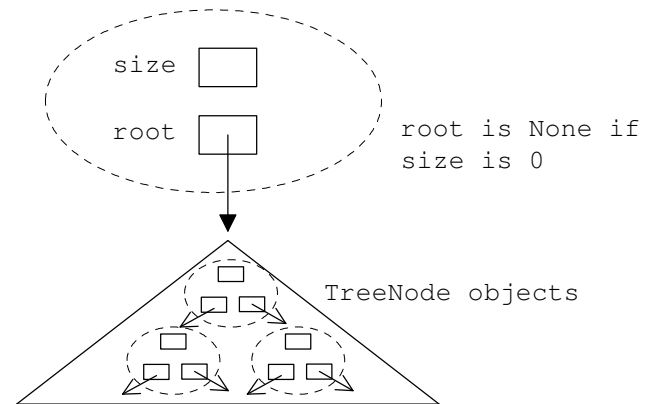
    def hasAnyChildren(self):
        return self.rightChild or self.leftChild

    def hasBothChildren(self):
        return self.rightChild and self.leftChild

    def replaceNodeData(self, key, value, lc, rc):
        self.key = key
        self.payload = value
        self.leftChild = lc
        self.rightChild = rc
        if self.hasLeftChild():
            self.leftChild.parent = self
        if self.hasRightChild():
            self.rightChild.parent = self

    def __iter__(self):
        if self:
            if self.hasLeftChild():
                for elem in self.leftChild:
                    yield elem
            yield self.key
            if self.hasRightChild():
                for elem in self.rightChild:
                    yield elem

```

A `BinarySearchTree` object

```

class BinarySearchTree:
    def __init__(self):
        self.root = None
        self.size = 0

    def length(self):
        return self.size

    def __len__(self):
        return self.size

    def __iter__(self):
        return self.root.__iter__()

    def __str__(self):
        """Returns a string representation of the tree
        rotated 90 degrees counter-clockwise"""

    def strHelper(root, level):
        resultStr = ""
        if root:
            resultStr += strHelper(root.rightChild,
                                   level+1)
            resultStr += "| " * level
            resultStr += str(root.key) + "\n"
            resultStr += strHelper(root.leftChild,
                                   level+1)

        return resultStr

    return strHelper(self.root, 0)

```

a) How do the `BinarySearchTree` `__iter__` and `__str__` methods work?

More partial `TreeNode` class and partial `BinarySearchTree` class.

```
class BinarySearchTree:
    ...
    def __contains__(self, key):
        if self._get(key, self.root):
            return True
        else:
            return False

    def get(self, key):
        if self.root:
            res = self._get(key, self.root)
            if res:
                return res.payload
            else:
                return None
        else:
            return None

    def _get(self, key, currentNode):
        if not currentNode:
            return None
        elif currentNode.key == key:
            return currentNode
        elif key < currentNode.key:
            return self._get(key, currentNode.leftChild)
        else:
            return self._get(key, currentNode.rightChild)

    def __getitem__(self, key):
        return self.get(key)

    def __setitem__(self, k, v):
        self.put(k, v)

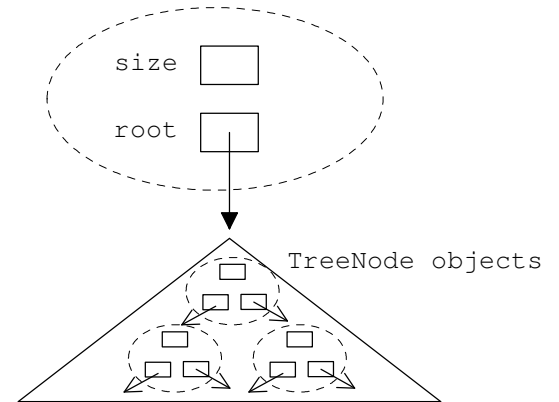
    def put(self, key, val):
        if self.root:
            self._put(key, val, self.root)
        else:
            self.root = TreeNode(key, val)
            self.size = self.size + 1

    def _put(self, key, val, currentNode):
        if key < currentNode.key:
            if currentNode.hasLeftChild():
                self._put(key, val, currentNode.leftChild)
            else:
                currentNode.leftChild = TreeNode(key, val, parent=currentNode)

        elif key > currentNode.key:
            if currentNode.hasRightChild():
                self._put(key, val, currentNode.rightChild)
            else:
                currentNode.rightChild = TreeNode(key, val, parent=currentNode)

        else:
            currentNode.payload = val
```

A `BinarySearchTree` object



b) The `_get` method is the "work horse" of BST search. It recursively walks `currentNode` down the tree until it finds `key` or becomes `None`. In English, what are the base and recursive cases?

c) What is the `put` method doing?

d) Complete the recursive `_put` method.

e) Draw the "shape" of the BST after `puts` of: 50, 60, 30, 70, 90, 40, 65

f) If "n" items are in the BST, what is `put`'s: Best-case $O(\quad)$? Worst-case $O(\quad)$? Average-case $O(\quad)$?

2. More partial TreeNode class and partial BinarySearchTree class.

```
class BinarySearchTree:
```

```
    ...
```

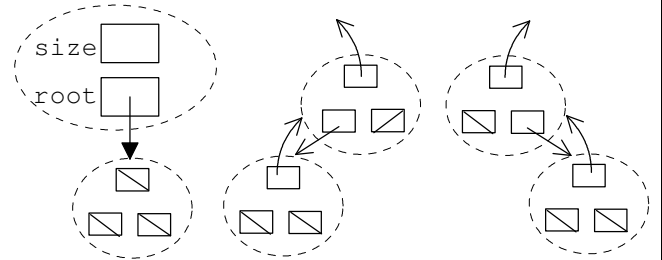
```
def delete(self, key):
    if self.size > 1:
        nodeToRemove = self._get(key, self.root)
        if nodeToRemove:
            self.remove(nodeToRemove)
            self.size = self.size - 1
        else:
            raise KeyError('Error, key not in tree')
    elif self.size == 1 and self.root.key == key:
        self.root = None
        self.size = self.size - 1
    else:
        raise KeyError('Error, key not in tree')
```

```
def __delitem__(self, key):
    self.delete(key)
```

```
def remove(self, currentNode):
    if currentNode.isLeaf(): #leaf
        if currentNode == currentNode.parent.leftChild:
            currentNode.parent.leftChild = None
        else:
            currentNode.parent.rightChild = None
    elif currentNode.hasBothChildren(): #interior
        succ = currentNode.findSuccessor()
        succ.spliceOut()
        currentNode.key = succ.key
        currentNode.payload = succ.payload
    else: # this node has one child
        if currentNode.hasLeftChild():
            if currentNode.isLeftChild():
                currentNode.leftChild.parent = currentNode.parent
                currentNode.parent.leftChild = currentNode.leftChild
            elif currentNode.isRightChild():
                currentNode.leftChild.parent = currentNode.parent
                currentNode.parent.rightChild = currentNode.leftChild
            else:
                currentNode.replaceNodeData(currentNode.leftChild.key,
                                             currentNode.leftChild.payload,
                                             currentNode.leftChild.leftChild,
                                             currentNode.leftChild.rightChild)
        else:
            if currentNode.isLeftChild():
                currentNode.rightChild.parent = currentNode.parent
                currentNode.parent.leftChild = currentNode.rightChild
            elif currentNode.isRightChild():
                currentNode.rightChild.parent = currentNode.parent
                currentNode.parent.rightChild = currentNode.rightChild
            else:
                currentNode.replaceNodeData(currentNode.rightChild.key,
                                             currentNode.rightChild.payload,
                                             currentNode.rightChild.leftChild,
                                             currentNode.rightChild.rightChild)
```

a) Update picture where we delete a leaf.

BinarySearchTree



b) Where in the code is each handled?

c) Draw all pictures deleting all nodes with one child.

3. Yet even more partial TreeNode class and partial BinarySearchTree class.

```
class TreeNode:
    ...
    def findSuccessor(self):
        succ = None
        if self.hasRightChild():
            succ = self.rightChild.findMin()
        else:
            if self.parent:
                if self.isLeftChild():
                    succ = self.parent
                else:
                    self.parent.rightChild = None
                    succ = self.parent.findSuccessor()
                    self.parent.rightChild = self
            return succ

    def findMin(self):
        current = self
        while current.hasLeftChild():
            current = current.leftChild
        return current

    def spliceOut(self):
        if self.isLeaf():
            if self.isLeftChild():
                self.parent.leftChild = None
            else:
                self.parent.rightChild = None
        elif self.hasAnyChildren():
            if self.hasLeftChild():
                if self.isLeftChild():
                    self.parent.leftChild = self.leftChild
                else:
                    self.parent.rightChild = self.leftChild
                    self.leftChild.parent = self.parent
            else:
                if self.isLeftChild():
                    self.parent.leftChild = self.rightChild
                else:
                    self.parent.rightChild = self.rightChild
                    self.rightChild.parent = self.parent
```