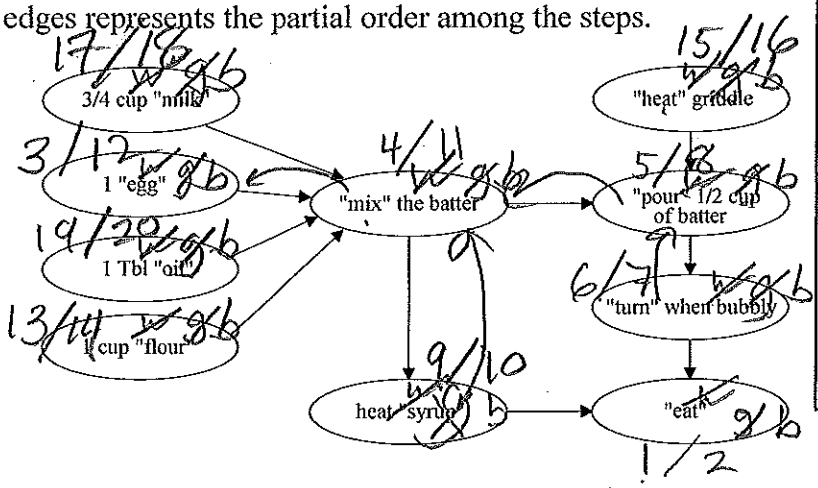
4. Section 7.5 uses recursion and the run-time stack to implement a DFS traversal. The `DFSGraph` uses a `time` attribute to note when a vertex if first encountered (`discovery` attribute) in the depth-first search and when a vertex in backtracked through (`finish` attribute). Consider the graph for making pancakes where vertices are steps and edges represents the partial order among the steps.




```
from graph import Graph
class DFSGraph(Graph):

    def __init__(self):
        super().__init__()
        self.time = 0

    def dfs(self):
        for aVertex in self:
            aVertex.setColor('white')
            aVertex.setPred(-1)
        for aVertex in self:
            if aVertex.getColor() == 'white':
                self.dfsvisit(aVertex)

    def dfsvisit(self,startVertex):
        startVertex.setColor('gray')
        self.time += 1
        startVertex.setDiscovery(self.time)
        for nextVertex in startVertex.getConnections():
            if nextVertex.getColor() == 'white':
                nextVertex.setPred(startVertex)
                self.dfsvisit(nextVertex)
        startVertex.setColor('black')
        self.time += 1
        startVertex.setFinish(self.time)
```

a) Assume (why is this a bad assumption???) that the for-loops alway iterate through the vertexes alphabetically (e.g., "eat", "egg", "flour", ...) by their id. **Write on the above graph the `discovery` and `finish` attributes assigned to each vertex by executing the `dfs` method.**

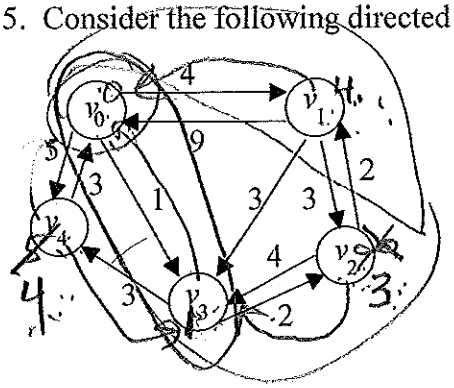
(see above)

b) A *topological sort* algorithm can use the dfs `discovery` and `finish` attributes to determine a proper order to avoid putting the "cart before the horse." For example, we don't want to "pour ½ cup of batter" before we "mix the batter", and we don't want to "mix the batter" until all the ingredients have been added. Outline the steps to perform a topological sort.

descending order of finish times

oil, milk, heat, flour, egg, mix, syrup, turn, eat

5. Consider the following directed graph (diagraph).



Dijkstra's Algorithm is a *greedy algorithm* that finds the shortest path from some vertex, say v_0, to all other vertices. A *greedy algorithm,* unlike divide-and-conquer and dynamic programming algorithms, DOES NOT divide a problem into smaller subproblems. Instead a greedy algorithm builds a solution by making a sequence of choices that look best ("locally" optimal) at the moment without regard for past or future choices (no backtracking to fix bad choices). Dijkstra's algorithm builds a subgraph by repeatedly selecting the next closest vertex to v_0 that is not already in the subgraph. Initially, only vertex v_0 is in the subgraph with a distance of 0 from itself.

a) What would be the order of vertices added to the subgraph during Dijkstra's algorithm?

v_0, v_3, v_2, v_1, v_4

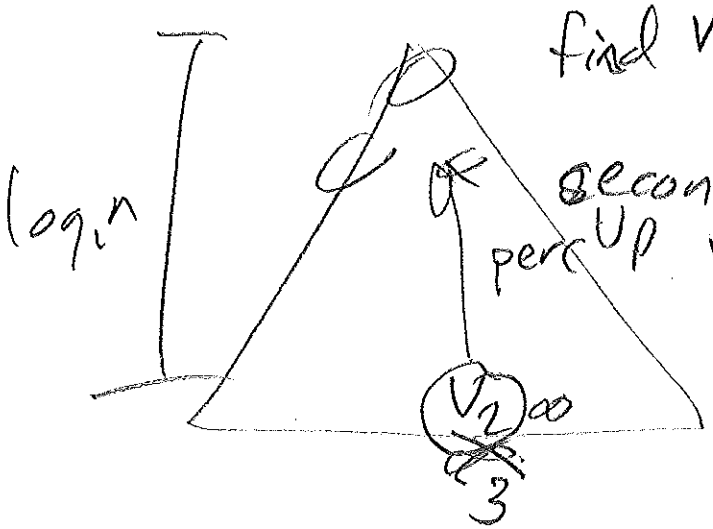
b) What *greedy criteria* did you use to select the next vertex to add to the subgraph?

Vertex not in the subgraph with smallest distance

c) What data structure could be used to efficiently determine that selection? min. heap

d) How might this data structure need to be modified? ← (lect. 27 page 3 decreaseKey function)

find v_2 first



The `PriorityQueue` class including the `__contains__` and `decreaseKey` methods:.

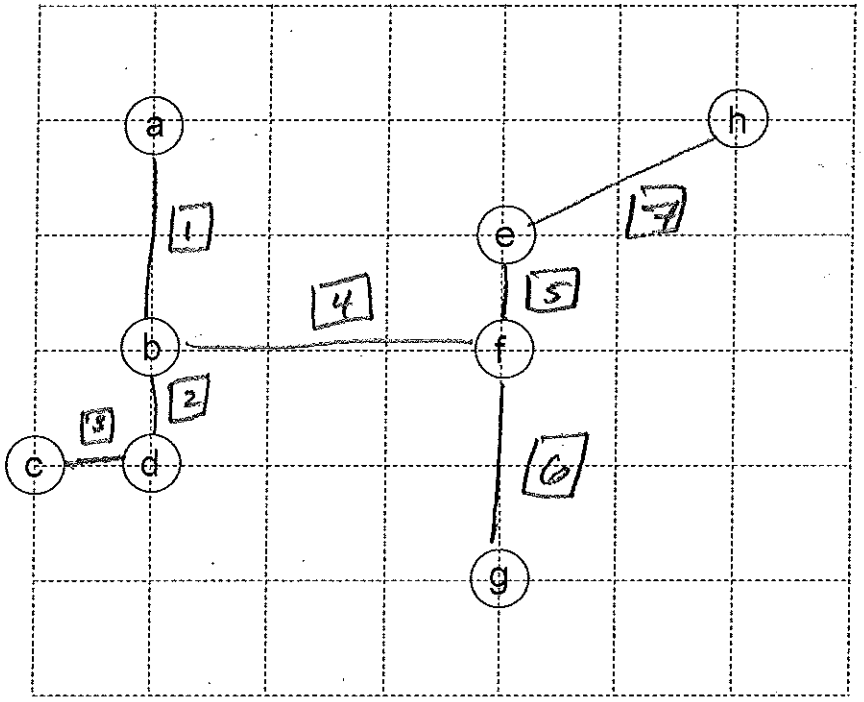
```
class PriorityQueue:
    def __init__(self):
        self.heapArray = [(0,0)]
        self.currentSize = 0

    def buildHeap(self,alist):
        self.currentSize = len(alist)
        self.heapArray = [(0,0)]
        for i in alist:
            self.heapArray.append(i)
        i = len(alist) // 2
        while (i > 0):
            self.percDown(i)
            i = i - 1

    def percDown(self,i):
        while (i * 2) <= self.currentSize:
            mc = self.minChild(i)
            if self.heapArray[i][0] > self.heapArray[mc][0]:
                tmp = self.heapArray[i]
                self.heapArray[i] = self.heapArray[mc]
                self.heapArray[mc] = tmp
            i = mc

    def minChild(self,i):
        if i*2 > self.currentSize:
            return -1
        else:
            if i*2 + 1 > self.currentSize:
                return i*2
            else:
                if self.heapArray[i*2][0] < self.heapArray[i*2+1][0]:
                    return i*2
                else:
                    return i*2+1

    def percUp(self,i):
        while i // 2 > 0:
            if self.heapArray[i][0] < self.heapArray[i//2][0]:
               tmp = self.heapArray[i//2]
               self.heapArray[i//2] = self.heapArray[i]
               self.heapArray[i] = tmp
            i = i//2

    def add(self,k):
        self.heapArray.append(k)
        self.currentSize = self.currentSize + 1
        self.percUp(self.currentSize)

    def delMin(self):
        retval = self.heapArray[1][1]
        self.heapArray[1] = self.heapArray[self.currentSize]
        self.currentSize = self.currentSize - 1
        self.heapArray.pop()
        self.percDown(1)
        return retval

    def isEmpty(self):
        if self.currentSize == 0:
            return True
        else:
            return False

    def decreaseKey(self,val,amt):
        # this is a little wierd, but we need to find the
        # heap thing to decrease by looking at its value
        done = False
        i = 1
        myKey = 0
        while not done and i <= self.currentSize:
            if self.heapArray[i][1] == val:
                done = True
                myKey = i
            else:
                i = i + 1
        if myKey > 0:
            self.heapArray[myKey] = (amt,self.heapArray[myKey][1])
            self.percUp(myKey)

    def __contains__(self,vtx):
        for pair in self.heapArray:
            if pair[1] == vtx:
                return True
        return False
```

3. If we want to speed-up the `__contains__` and `decreaseKey` methods, then what type of data structure could we add to aid in: checking for the existence of a key value, and if a key value exists in the heap, then at what index does it reside?

4. What modifications would need be needed to other methods to keep the data structure in question 3 up-to-date?

1. Suppose you had a map of settlements on the planet X
(Assume edges could connecting all vertices with their Euclidean distances as their costs)

[i] = added order



We want to build roads that allow us to travel between any pair of cities. Because resources are scarce, we want the total length of all roads build to be minimal. Since all cities will be connected anyway, it does not matter where we start, but assume we start at "a".

a) Assuming we start at city "a" which city would you connect first? Why this city?

(b) because it's closest to (a)

b) What city would you connect next to expand your partial road network?

(d) because it is closest to partial road system

c) What would be some characteristics of the resulting "graph" after all the cities are connected?

d) Does your algorithm come up with the overall best (globally optimal) result?