3. **General "Algorithmic-Complexity Analysis" terminology:**

<u>problem</u> - question we seek an answer for, e.g., "What is the sum of all the items in a list/array?"

<u>parameters</u> - variables with unspecified values

<u>problem instance</u> - assignment of values to parameters, i.e., the specific input to the problem

	0	1	2	3	4	5	6
myList:	5	10	2	15	20	1	11

sum: ?

(number of elements) n: 7

<u>algorithm</u> - step-by-step procedure for producing a solution

<u>basic operation</u> - fundamental operation in the algorithm (i.e., operation done the most) Generally, we want to derive a function for the number of times that the basic operation is performed related to the *problem size.*

<u>problem size</u> - input size. For algorithms involving lists/arrays, the problem size is the number of elements ("n").

<u>Big-oh notation ($O(\,)$)</u> - As the size of a problem grows (i.e., more data), how will our program's run-time grow.

Consider the following `sumList` function.

```
def sumList(myList):
    """Returns the sum of all items in myList"""
    total = 0
    for item in myList:
        total = total + item
    return total
```

a) What is the basic operation of `sumList` (i.e., operation done the most) ? addition

b) What is the problem size of `sumList`? len of myList, "n"

c) If n is 10000 and `sumList` takes 10 seconds, how long would you expect `sumList` to take for n of 20000?

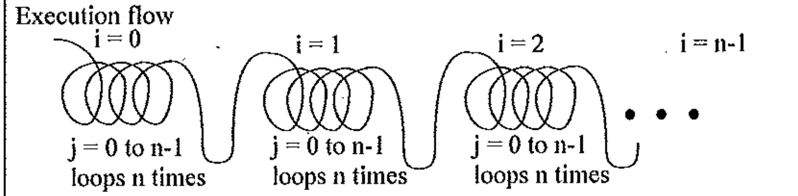
20 sec

d) What is the big-oh notation for `sumList`? $O(n)$ "linear"

4. Consider the following `someLoops` function.

```
def someLoops(n):
    total = 0
    for i in range(n):
        for j in range(n):
            total = total + i + j
    return total
```

Execution flow



a) What is the basic operation of `someLoops` (i.e., operation done the most)? addition

inner-most loop body

b) How many times will the basic operation execute as a function of n?

$n \times n = n^2$

c) What is the big-oh notation for `someLoops`? $O(n^2)$ "quadratic"

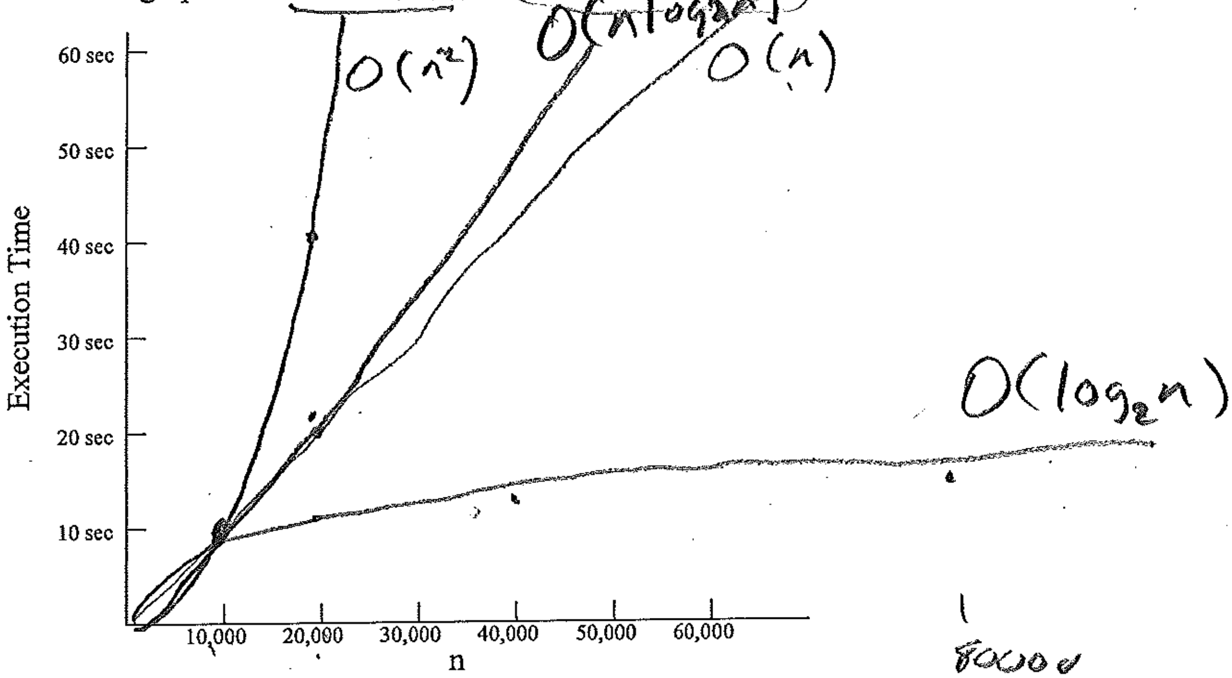
d) If we input n of 10000 and `someLoops` takes 10 seconds, how long would you expect `someLoops` to take for n of 20000?

exec. time $T(n) = c + c_1 n + \underline{c_2 n^2}$ $T(10000) \approx c\, 10000^2 = 10 sec$

$O(n^2)$ $T(n) \approx c n^2$ $c = \frac{10 sec}{10000^2}$

$T(20000) = c\, 20000^2 = \frac{10 sec}{10000^2} \cdot 20000^2 = 40 sec$

1. Draw the graph for `sumList` (O(n)) and `someLoops` (O(n^2)) from the previous lecture.



2. Consider the following `sumSomeListItems` function.

```
import time

def main():
    n = eval(input("Enter size of list: "))
    aList = list(range(1, n+1))
    start = time.perf_counter()   # <<<< time.clock() is deprecated
    sum = sumSomeListItems(aList)
    end = time.perf_counter()     # <<<< time.clock() is deprecated
   print("Time to sum the list was %.9f seconds" % (end-start))

def sumSomeListItems(myList):
    """Returns the sum of some items in myList"""
    total = 0
    index = len(myList) - 1
    while index > 0:
        total = total + myList[index]
        index = index // 2
    return total

main()
```

0 1 2 3 ... n-1

| 1 | 2 | 3 | 4 | 5 | 6 | ... | n |

32K 16K 8K 4K 2K 1K 1024 n = 512, 256, 128, 64 32 16 8 4 2 1

2^{15} 2^{14} 2^{13} 2^{12} 2^{11} 2^{10} 2^9 2^8 2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

$\log_2 n = x$ iff $2^x = n$

$\log_2 x = \dfrac{\log_{10} x}{\log_{10} 2}$

a) What is the problem size of `sumSomeListItems`? n

b) If we input n of 10,000 and `sumSomeListItems` takes 10 seconds, how long would you expect `sumSomeListItems` to take for n of 20,000?

(Hint: For n of 20,000, how many more times would the loop execute than for n of 10,000?)

$O(\log_2 n)$ $T(n) = c \log_2 n$ $T(10000) = c \log_2 10000 = 10 \text{ sec}$ ($\log_2 10000 \approx 13.x$)

$c = \dfrac{10 \text{ sec}}{13.x}$

$T(20000) = c \log_2 20000 = \left(\dfrac{10 \text{ sec}}{13.x}\right) 14.x$

c) What is the big-oh notation for `sumSomeListItems`? $O(\log_2 n)$

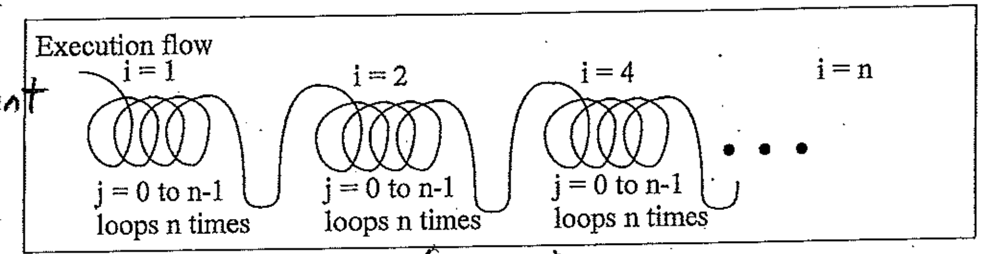
d) Add the execution-time graph for `sumSomeListItems` to the graph.

3.

```
i = 1
while i <= n:
    for j in range(n):
        # something of O(1)
    # end for
    i = i * 2
# end while
```

constant

"basic op."



a) Analyze the above algorithm to determine its big-oh notation, $O()$. $(\log_2 n) \times n$

$O(n \cdot \log_2 n)$

b) If n of 10,000, takes 10 seconds, how long would you expect the above code to take for n of 20,000?

$T(n) = c\, n \log_2 n \quad T(10000) = c\, 10000 \log_2 10000 = 10\text{ sec}$

$c = \dfrac{10\text{ sec}}{10000 \log_2 10000} = \dfrac{10\text{ sec}}{10000 \cdot 13.}$

$T(20000) = c\, 20000 \log_2 20000 = ?\text{ sec}$

$= \dfrac{10\text{ sec}}{\cancel{10000} \times 13.x} \, 2\cancel{0000} \underbrace{\log_2 20000}_{14.x} \approx 20.z$

c) Add the execution-time graph for the above code to the graph.

4. Most programming languages have a built-in array data structure to store a collection of same-type items. Arrays are implemented in RAM memory as a contiguous block of memory locations. Consider an array X that contains the odd integers:

address	RAM Memory (byte) — 8 bit	
4000	1	X[0]
4004	3	X[1]
4008	5	X[2]
4012	7	X[3]
4016	9	X[4]
4020	11	X[5]
4024	13	X[6]
⋮		

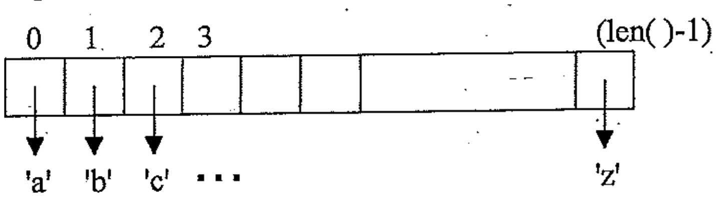
a) Any array element can be accessed randomly by calculating its address. For example, address of X[5] = 4000 + 5 * 4 = 4020. What is the general formula for calculating the address of the ith element in an array?

$X[i] = (\text{start addr. of } X) + i * (\text{element size})$

b) What is the big-oh notation for accessing the ith element?

$O(1)$

c) A Python list uses an array of references (pointers) to list items in their implementation of a list. For example, a list of strings containing the alphabet:



Since a Python list can contain heterogeneous data, how does storing references in the list aid implementation?

ref. are same size so O(1) access to any list item by index

5. Arrays in most HLLs are static in size (i.e., cannot grow at run-time), so arrays are constructed to hold the "maximum" number of items. For example, an array with 1,000 slots might only contain 3 items:

size: 3

	0	1	2	3				999
scores:	20	10	30	40				

5 20 10 30 40

a) The *physical size* of the array is the number of slots in the array. What is the physical size of scores? 1000

b) The *logical size* of the array is the number of items actually in the array. What is the logical size of scores? 3 "n"

c) The *load factor* is faction of the array being used. What is the load factor of scores? 3/1000 = 0.003 0.3%

d) What is the $O()$ for "appending" a new score to the "right end" of the array? $O(1)$

e) What is the $O()$ for adding a new score to the "left end" of the array? $O(n)$ n = logical size

f) What is the *average* $O()$ for adding a new score to the array? $O(\frac{n}{2}) = O(n)$

g) During run-time if an array fills up and we want to add another item, the program can usually:

- Create a bigger array than the one that filled up
- Copy all the items from the old array to the bigger array
- Add the new item
- Delete the smaller array to free up its memory

When creating the bigger array, how much bigger than the old array should it be? double size of old one

h) What is the $O()$ of moving to a larger array? $O(n)$

6. Consider the following list methods in Python:

Method	Usage	Average $O()$ for myList containing n items
index []	`itemValue = myList[i]`	$O(1)$
	`myList[i] = newValue`	$O(1)$
append	`myList.append(item)`	$O(1)$
~~extend~~	~~`myList.extend(otherList)`~~	
insert	`myList.insert(i, item)`	$O(n/2)$
pop	`myList.pop( )`	$O(1)$
pop(i)	`myList.pop(i)`	$O(n/2)$
del	`del myList[i]`	$O(n/2)$
remove	`myList.remove(item)`	$O(n)$
index	`myList.index(item)`	$O(n)$
iteration	`for item in myList:`	$O(n)$
reverse	`myList.reverse( )`	$O(n)$

Dictionary Operations:

Method	Usage	Explanation	Average $O()$ for n keys
get item	`myDictionary.get(myKey)` `value = myDictionary[myKey]`	Returns the value associated with myKey; otherwise *None*	$O(1)$
set item	`myDictionary[myKey]=value`	Change or add myKey:value pair	$O(1)$
in	`myKey in myDictionary`	Returns True if myKey is in myDictionary; otherwise False	$O(1)$
del	`del myDictionary[myKey]`	Deletes the mykey:value pair	$O(1)$