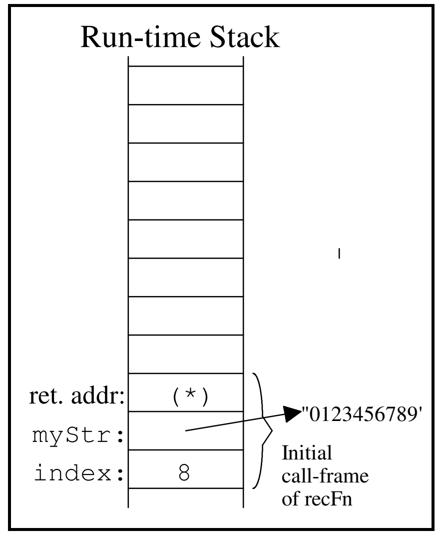
Spring 2015 Name: ______________________

Data Structures - Test 2

Question 1. (10 points) What is printed by the following program? Output:

```
def recFn(myStr, index):
   print(myStr[index], index)
   if index < 4:
      return "Hi"
    else:
      return recFn(myStr, index - 3) + myStr[index]
                 (**)


print("result =",recFn("0123456789", 8))
                    (*)
```



Question 2. Write a recursive Python function to calculate a^n (where n is an integer) based on the formulas:

$a^0 = 1$, for n = 0

$a^1 = a$, for n = 1

$a^n = a^{n/2} a^{n/2}$, for even n > 1 (recall we can check for this in Python by `n % 2 == 0`)

$a^n = a^{(n-1)/2} a^{(n-1)/2} a$, for odd n > 1

a) (12 points) Complete the below `powerOf` **recursive** function

```
def powerOf(a, n):
```

b) (8 points) For the above recursive `powerOf` function, complete the calling-tree for `powerOf` (2, 6).

```
                 powerOf(2,6)
      powerOf(2,3)            powerOf(2,3)
```

c) (5 points) Suggest a way to speedup the above `powerOf` function.

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Question 3. Consider the following insertion sort which sorts in ascending order, but builds the sorted part on the right.

```
def insertionSort(myList):
    myListLength = len(myList)
    for lastUnsortedIndex in range(len(myList)-2, -1, -1):
        itemToInsert = myList[lastUnsortedIndex]
        testIndex = lastUnsortedIndex + 1
        while testIndex < myListLength and myList[testIndex] < itemToInsert:
            myList[testIndex-1] = myList[testIndex]
            testIndex = testIndex + 1
        myList[testIndex - 1] = itemToInsert
```

a) (5 points) What is the purpose of the `testIndex < myListLength` while-loop comparison?

Consider the modified insertion sort code that eliminates the `testIndex < myListLength` while-loop comparison, but adds the **bold** code.

```
def insertionSortB(myList):
    myList.append(max(myList))
    for lastUnsortedIndex in range(len(myList)-2, -1, -1):
        itemToInsert = myList[lastUnsortedIndex]
        testIndex = lastUnsortedIndex + 1
        while myList[testIndex] < itemToInsert:
            myList[testIndex-1] = myList[testIndex]
            testIndex = testIndex + 1
        myList[testIndex - 1] = itemToInsert
    myList.pop()
```

b) (5 points) Explain how the **bolded** code in the modified insertion sort code above allows for the elimination of the `testIndex < myListLength` while-loop comparison.

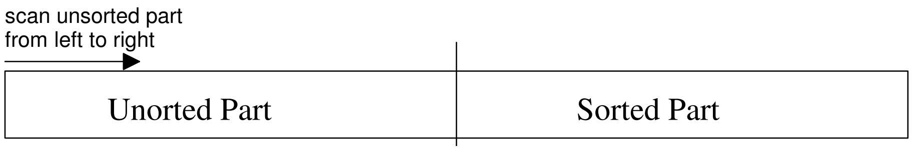
Consider the following timing of the above two insertion sorts on lists of 10000 elements.

Initial arrangement of list before sorting	`insertionSort` - at the top of page	`insertionSortB` - modified version in middle of the page
Sorted in descending order: 10000, 9999, ..., 2, 1	14.1 seconds	12.6 seconds
Already in ascending order: 1, 2, ..., 9999, 10000	0.004 seconds	0.004 seconds
Randomly ordered list of 10000 numbers	7.3 seconds	6.5 seconds

c) (5 points) Explain why `insertionSortB` (modified version in middle of page) out performs the original `insertionSort`.

d) (5 points) In either version, why does sorting the randomly order list take about halve the time of sorting the initially descending ordered list?

Question 4. In class we discussed the following bubble sort code which sorts in ascending order (smallest to largest) and builds the sorted part on the right-hand side of the list, i.e.:

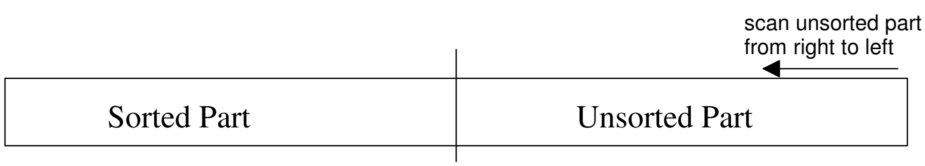



```
def bubbleSort(myList):
    for lastUnsortedIndex in range(len(myList)-1,0,-1):
        # scan the unsorted part at the beginning of myList
        for testIndex in range(lastUnsortedIndex):

            # if we find two adjacent items out of ascending order, then switch them
            if myList[testIndex] > myList[testIndex+1]:
                temp = myList[testIndex]
                myList[testIndex] = myList[testIndex+1]
                myList[testIndex+1] = temp
```

(20 points) For this question write a variation of the above bubble sort that:

- sorts in **descending order** (largest to smallest)
- builds the **sorted part on the left-hand side** of the list, i.e.,




```
def bubbleSortVariation(myList):
```

Question 5. Recall the common rehashing strategies we discussed for open-address hashing:

Strategy	Description
quadratic probing	Check the square of the attempt-number away for an available slot, i.e., [home address + ((rehash attempt #)^2 +(rehash attempt #))/2] % (hash table size), where the hash table size is a power of 2. Integer division is used above

a) (8 points) Insert "Paul Gray" and then "Sarah Diesburg" using Linear (on left) and Quadratic (on right) probing.

Hash Table with Linear Probing

0	Ben Schafer
1	
2	
3	Philip East
4	
5	
6	Mark Fienup
7	John Doe

Hash function

hash(John Doe) = 7

hash(Philip East) = 3

hash(Mark Fienup) = 6

hash(Ben Schafer) = 0

hash(Paul Gray) = 3

hash(Sarah Diesburg) = 3

Hash Table with Quad. Probing

0	Ben Schafer
1	
2	
3	Philip East
4	
5	
6	Mark Fienup
7	John Doe

b) (7 points) Explain why both linear and quadratic probing both suffer from primary clustering?

Question 6. (10 points) The general idea of *Quick sort* is as follows:

- Select a "random" item in the unsorted part as the *pivot*
- Rearrange (*partition*) the unsorted items as shown in diagram on right:
- Quick sort the unsorted part to the left of the pivot
- Quick sort the unsorted part to the right of the pivot

Pivot Index

All items < to Pivot	Pivot Item	All items >= to Pivot

Explain why the **worst-case** performance is $\boldsymbol{O}(\mathbf{n}^2)$.