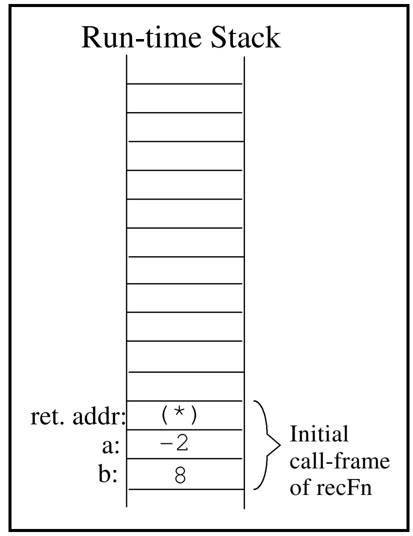
 Name: ______________________

Data Structures - Test 2

Question 1. (10 points) What is printed by the following program? **Output:**

```
def recFn(a, b):
    print( a, b )
    if  a == b:
        return b
    elif a > b:
        return a
    else:
        return a + recFn(a + 2, b - 2) + b
                            (**)

print("Result = ", recFn(-2, 8))
                            (*)
```



Question 2. Write a recursive Python function to calculate a^n (where n is an integer) based on the formulas:

$a^0 = 1$, for n = 0

$a^1 = a$, for n = 1

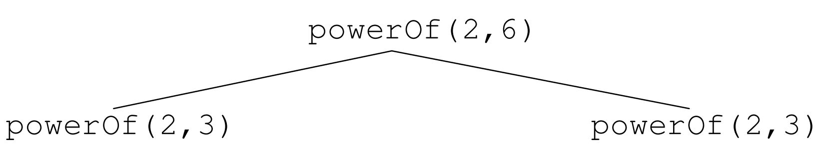
$a^n = a^{n/2}a^{n/2}$, for even n > 1 (recall we can check for this in Python by `n % 2 == 0`)

$a^n = a^{(n-1)/2}a^{(n-1)/2}a$, for odd n > 1

a) (8 points) Complete the below `powerOf` **recursive** function

```
def powerOf(a, n):
```

b) (7 points) For the above recursive `powerOf` function, complete the calling-tree for `powerOf` (2, 6).



c) (5 points) Suggest a way to speedup the above `powerOf` function.

Question 3. (16 points) Consider the following simple sorts discussed in class -- all of which sort in ascending order.

```
def bubbleSort(myList):
    for lastUnsortedIndex in range(len(myList)-1,0,-1):
        for testIndex in range(lastUnsortedIndex):
            if myList[testIndex] > myList[testIndex+1]:
                temp = myList[testIndex]
                myList[testIndex] = myList[testIndex+1]
                myList[testIndex+1] = temp
```

```
def insertionSort(myList):
    for firstUnsortedIndex in range(1,len(myList)):
        itemToInsert = myList[firstUnsortedIndex]
        testIndex = firstUnsortedIndex - 1
        while testIndex >= 0 and myList[testIndex] > itemToInsert:
            myList[testIndex+1] = myList[testIndex]
            testIndex = testIndex - 1

        myList[testIndex + 1] = itemToInsert
```

```
def selectionSort(aList):
    for lastUnsortedIndex in range(len(aList)-1, 0, -1):
        maxIndex = 0
        for testIndex in range(1, lastUnsortedIndex+1):
            if aList[testIndex] > aList[maxIndex]:
                maxIndex = testIndex
        # exchange the items at maxIndex and lastUnsortedIndex
        temp = aList[lastUnsortedIndex]
        aList[lastUnsortedIndex] = aList[maxIndex]
        aList[maxIndex] = temp
```

Timings of Above Sorting Algorithms on 10,000 items (seconds)			
Type of sorting algorithm	Initial Ordering of Items		
	Descending	Ascending	Random order
bubbleSort.py	23.3	7.7	15.8
insertionSort.py	14.2	0.004	7.3
selectionSort.py	7.1	7.7	6.8

a) Explain why insertionSort on a descending list (14.2 s) takes about **twice as long** as insertionSort on a random list (7.3 s).

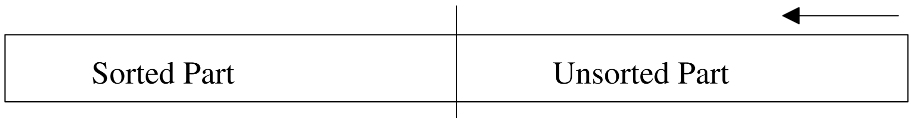
b) Explain why bubbleSort on a descending list (23.3 s) takes longer than insertionSort on a descending list (14.2 s).

c) Explain why selectionSort is $O(n^2)$ in the worst-case, where n is the size of the list being sorted.

Question 4. (20 points) In class we discussed the bubbleSort code shown in question 3 on page 2 which sorts in ascending order (smallest to largest) and builds the sorted part on the right-hand side of the list.

For this question write a variation of bubble sort that:

- sorts in **ascending order** still (smallest to largest), **but**
- adds a **check to stop early** if no swap occurs when scanning the unsorted part of the array, AND
- builds the **sorted part on the left-hand side** of the list, i.e.,



Inner loop scans from right to left across the unsorted part swapping adjacent items that are "out of order"

```
def bubbleSortVariation(myList):
```

Question 5. Recall the general idea of Quick sort:

- Partition by selecting a pivot item at "random" and then rearrange (*partitioning*) the unsorted items such that::
- Quick sort the unsorted part to the left of the pivot
- Quick sort the unsorted part to the right of the pivot

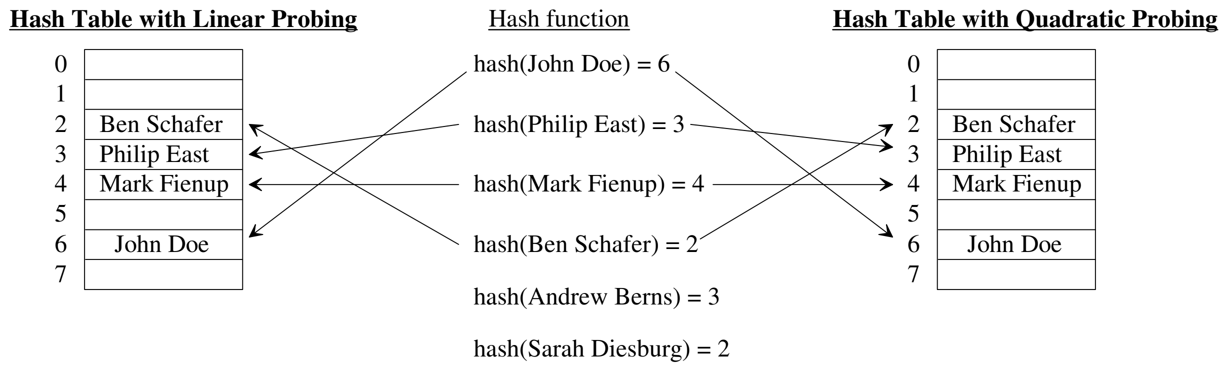
	Pivot Index	
All items < to Pivot	Pivot Item	All items >= to Pivot

(10 points) Explain why quick sort is O(n $\log_2$ n) when sorting initially randomly ordered items. (n is the len(myList))

Question 5. Two common rehashing strategies for open-address hashing are linear probing and quadratic probing:

quadratic probing	Check the square of the attempt-number away for an available slot, i.e., [home address + ((rehash attempt #)^2 +(rehash attempt #))//2] % (hash table size), where the hash table size is a power of 2. Integer division is used above

a) (8 points) Insert “Andrew Berns” **and then** “Sarah Diesburg” using Linear (on left) and Quadratic (on right) probing.



b) (8 points) Open-address hashing above, uses rehashing (e.g., linear or quadratic probing) when collisions occur. Initially, we used None to indicate that a hash table slot is "empty" and True to indicate that a slot had a "deleted" value. Explain why empty and deleted slots are treated differently.

c) (8 points) Briefly describe how closed-address hashing (e.g., ChainingDict) handles deletions.