Handling "Hard" Problems: For many optimization problems (e.g., TSP, knapsack, job-scheduling), the best known algorithms have run-time's that grow exponentially. Thus, you could wait centuries for the solution of all but the smallest problems!

Ways to handle these "hard" problems:

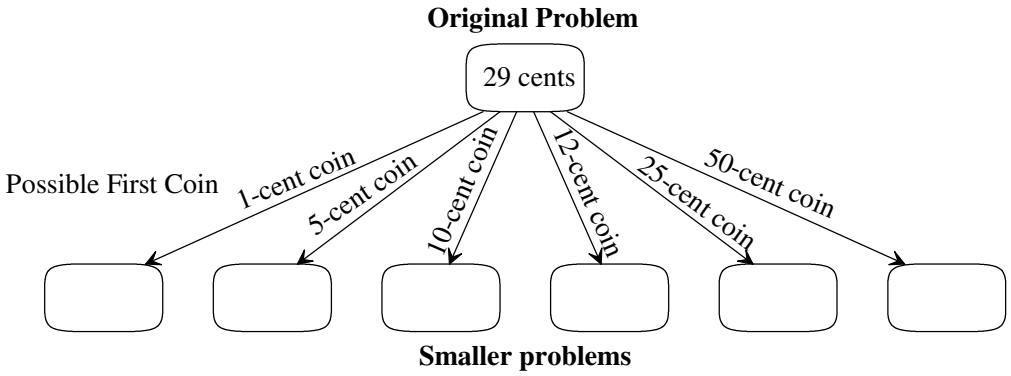
- Find the best (or a good) solution "quickly" to avoid considering the vast majority of the 2^n worse solutions, e.g, Backtracking (Chapter 5) and Branch-and-Bound (Chapter 6)
- See if a restricted version of the problem meets your needed that might have a tractable (polynomial, e.g., $\Theta(n^3)$) solution. e.g., Fractional Knapsack problem, TSP problem satisfying the triangle inequality
- Use an approximation algorithm to find a good, but not necessarily optimal solution

Backtracking (chapter 5) general idea:

- Search the "*state-space tree*" using depth-first search to find a suboptimal solution quickly
- Use the best solution found so far to prune partial solutions that are not "promising,", i.e., cannot lead to a better solution than one already found.

The goal is to prune enough of the state-space tree (exponential is size) that the optimal solution can be found in a reasonable amount of time. However, in the worst case, the algorithm is still exponential.

1. Consider an instance of the coin-change problem: For coin types of {1, 5, 10, 12, 25, 50} and a change amount of 29-cents, determine the **fewest number** of coins for this amount of change.



a) Draw a little more of the recursive state-space tree.

b) How would you recognize a solution?

c) What would be the first solution found with a depth-first search of this state-space tree?

d) Why does the order of coins make such a big difference in the number of nodes in the state-space tree?

Order of Coins	Change Amount 29-cents			Change Amount 399-cents		
	Fewest # coins	Execution time (sec.)	# Nodes in State-space	Fewest # coins	Execution time (sec.)	# Nodes in State-space
1 5 10 12 25 50	3	0	149	10	1,290	302,715,760
50 25 12 10 5 1	3	0	10	10	8	2,015,539

The general recursive backtracking algorithm for optimization problems (e.g., TSP, knapsack, job-scheduling) looks something like:

```
Backtrack( recursionTreeNode p ) {

    treeNode c;
    for each child c of p do                              # each c represents a possible choice
        if promising(c) then                              # c is "promising" if it could lead to a better solution
            if c is a solution that's better than best then   # check if this is the best solution found so far
                best = c                                  # remember the best solution
            else
                Backtrack(c)                              # follow a branch down the tree
            end if
        end if
    end for
} // end Backtrack
```

General Notes about Backtracking:

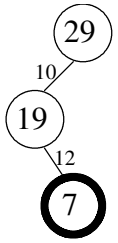
- The depth-first nature of backtracking only stores information about the current branch being explored so the memory usage is "low"
- Each node of the state-space (recursive-call) tree maintains the state of a partial solution. In general the partial solution state consists of potentially large arrays that change little between parent and child. To avoid having multiple copies of these arrays, a single "global" state is maintained which is updated before we go down to the child (via a recursive call) and undone when we backtrack to the parent.
- We could use the concept of backtracking without recursion by using a stack to maintain a collection of unexplored choices. Thus, we would simulate the run-time stack to drive the backtracking algorithm.

2. Consider customizing the above Backtrack template for the coin-change problem.

a) What would the "for each child c" loop iterate over? (What problem instance information is needed?)

b) What two criteria can be used to determine if a child node (c) is NOT promising?

c) What state information is needed at each node?



d) What information is needed by our promising function?