Consider the grid of rows of hexagonal cells below, and notice that in each row, each cell has two neighbors in the previous row. The *lookup table* in the upper-right corner shows the rule that the cells in each row follow to decide whether the neighbor below and between them will be shaded or left blank. The shaded cell in the top row is called the *seed,* or starting value, for the cellular automata. Working one row at a time, use the lookup table to shade the appropriate cells in each row.

How can you describe the rule in the lookup table in words? While you shade the cells in each row, look for developing patterns, but follow the lookup table strictly, continuing to work row by row. When you get to the last row, be prepared to describe and share your results.
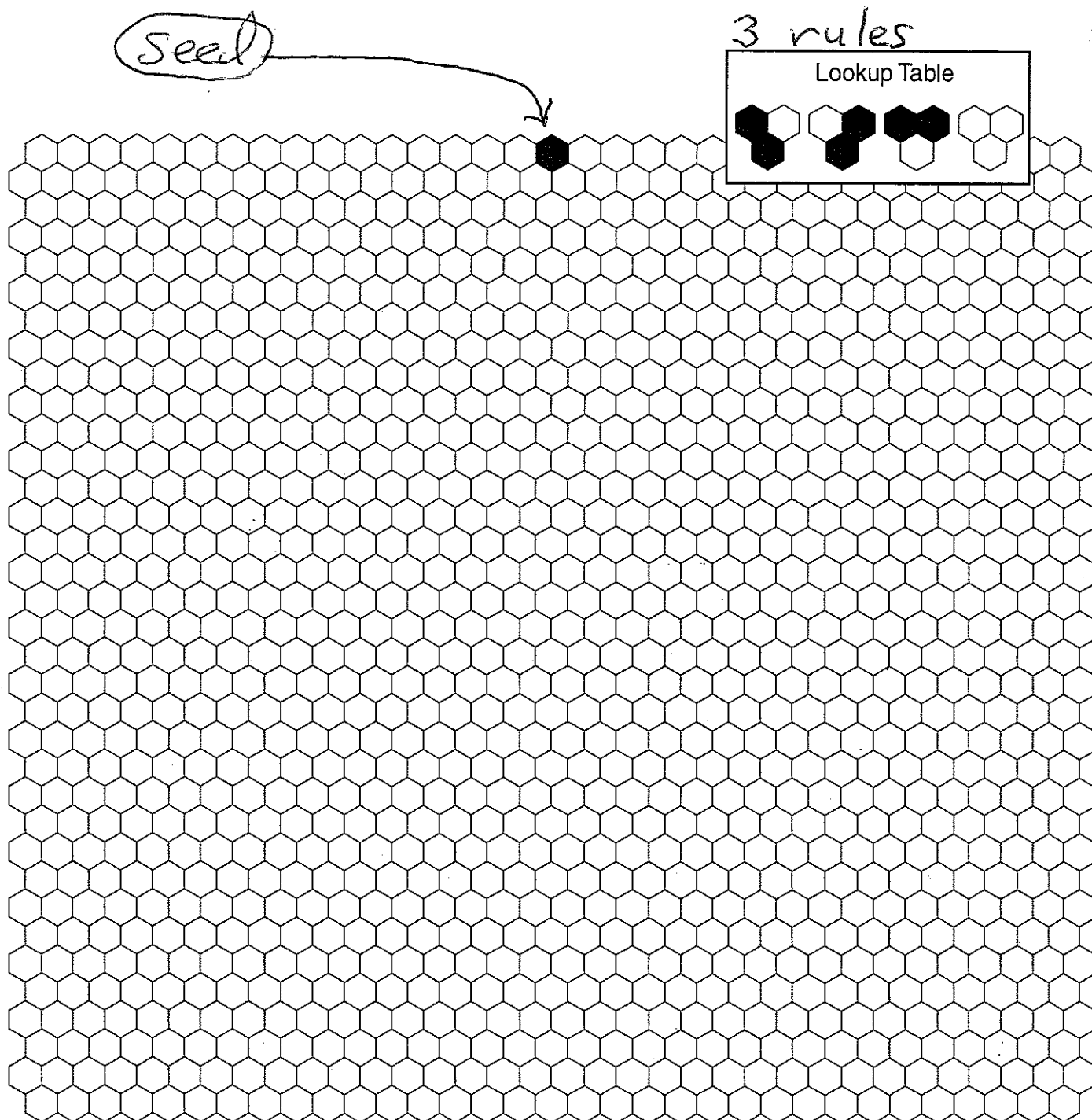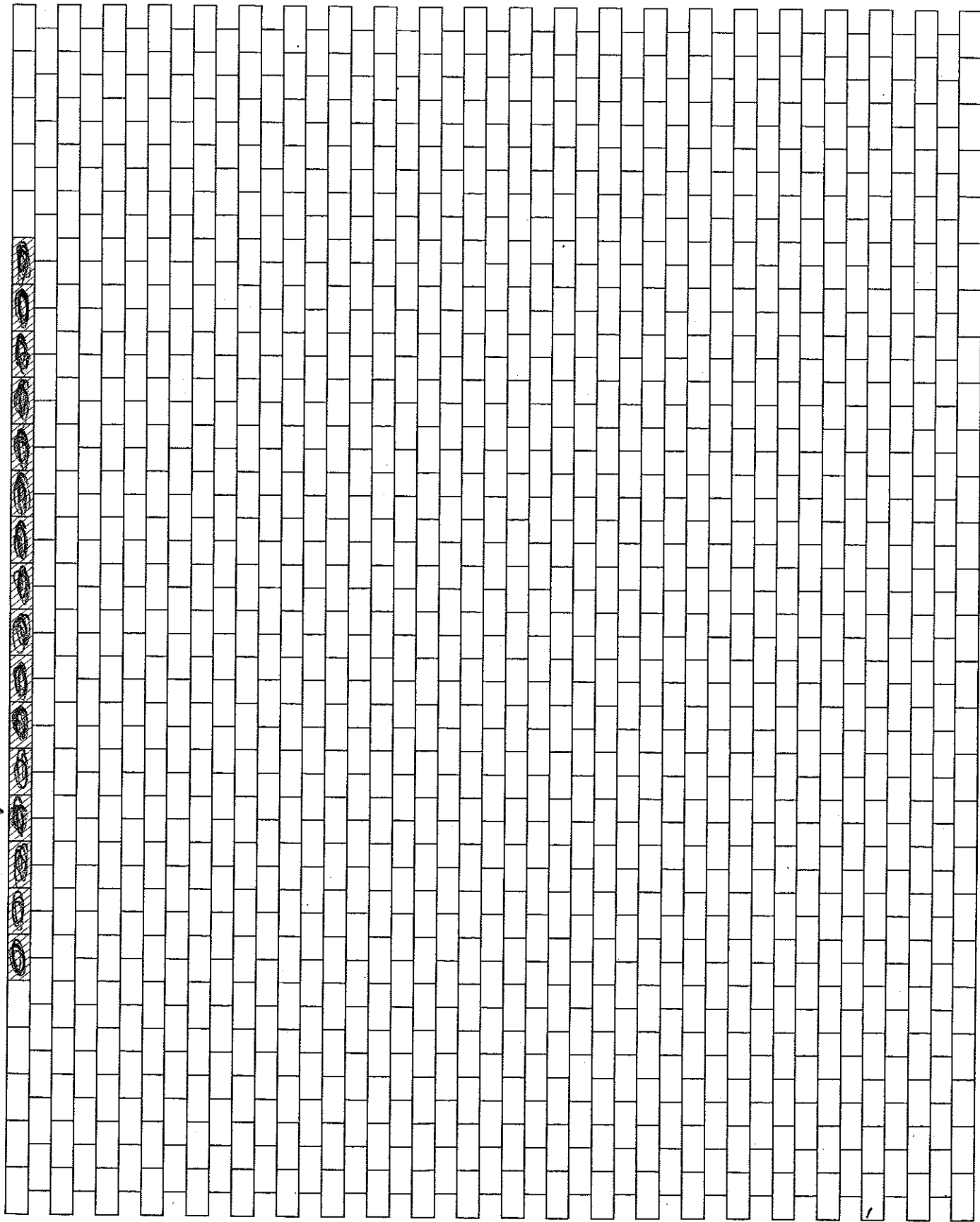
CELLULAR AUTOMATA: AN INTRODUCTION FOR STUDENTS

What do the burn patterns of forest fires, the percolation patterns of pollutants seeping into the earth, and the area over which a disease spreads have in common? All are complex images that can be modeled mathematically using simple rules and a dynamic process called *iteration*. In the past few years, the study of chaos theory has led scientists from many fields to conclude that even the most chaotic and random-seeming images contain underlying structure. Major breakthroughs have occurred in such widely varied areas as predicting the onset of heart attacks; understanding the stock market; compressing digital images for transmission; creating extremely complex computer images of trees and flowers; and actually modeling a computer version of artificial life, complete with mutations.

One process for generating and understanding some of these models is called *cellular automata* (pronounced "aw-TAH-ma-ta"), or *CAs* for short, the graphical equivalent of a fleet of tiny, simple computer programs, each capable of performing the same single simple task. These programs are executed in the form of cells, arranged on a grid, that apply a rule to determine whether their neighbors will be shaded or blank. CAs were first used as a theoretical tool in the late 1940s by such mathematicians as John Von Neumann; beginning in the late 1960s, CAs began to find their way into practical applications, such as those in these activities. When arranged in a series of lines or rows, these cells can represent generations of simple life forms. In two dimensions, CAs can also model the evolution of life forms, as in John Conway's "game of life," as well as such wildly diverse phenomena as population growth in amoebas, urban sprawl, or the spreading patterns of cancer cells or patches of wildflowers.

The following set of activities gives you an opportunity to explore the properties of CAs in both one and two dimensions. By using probability to introduce chaos into the process, you can produce and analyze your own models of forest-fire-burn patterns and oil-spill-percolation patterns. These activities will help you understand how to use mathematics to produce meaningful models of natural phenomena, to discuss and analyze these models, and to develop conjectures and reach conclusions. If you wish, you can further pursue this area on your own. Many resources that discuss and investigate CAs and mathematical modeling are available.

oil spill percolation model

A variation of CAs involves adding a little chaos to the process, by having the rule work only with a certain probability. For this model of an oil spill—as seen from the side, looking through the earth—the cells look more like bricks. Each cell in a row has two neighbors in the row below it, one to the left and one to the right. A shaded cell represents oil having spread, or percolated, past a soil particle; the particles themselves are the intersections of a cell and its neighbor below. The rule for this model is this: If a cell is shaded, a certain probability exists that it will shade each of its neighbors. Any cell could therefore have two chances of being shaded. If the cell gets shaded by its left neighbor above, no need exists to test the right one.

You will need a partner for this activity. Your teacher will assign you a probability between 0.5 and 0.75, along with instructions on how to decide whether a cell will shade each of its neighbors. Working together, you and your partner will make a model of a percolating oil spill. If your model runs off the paper, use another copy of this sheet to continue the model. How does this model differ from your previous ones? How is it similar? Compare your model with those of other students. How does yours compare with theirs? What relationship can you see between the probability used and the model's appearance?

TABLE OF RANDOM DIGITS — SHEET 3A

*=INT(RAND()*100)*
in Excel works too.

Table of random digits.