Computational Modeling and Simulation

CS 1025

Carrying Capacity. 1

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Let's start with a very common modeling problem: Population growth. This is a wonderful model to look at because it generalizes to so many other models. The model of population growth is analogous to financial savings and investments with fixed interest rates, labor productivity, and many others.

The premise of the population growth model is built upon our fundamental equation of "HAVE=HAD+CHANGE". What you HAVE in the current population is what you HAD the last time that you checked, plus the CHANGE which encompasses new births or deaths in the population.

The most straight forward interpretation of CHANGE is that the number of births in the population is proportional to the number of people in the population.

One way to think about this is to look at some data.

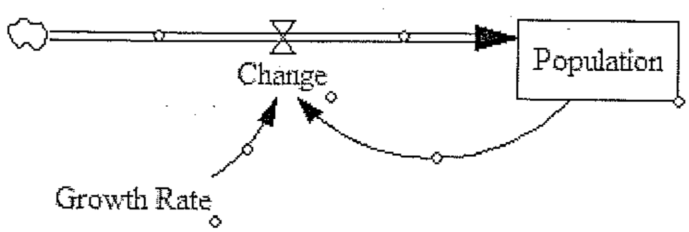
According to the CIA, the birth rate in China is 1.37%. A 1.37% growth rate means that in a group of 1000 people there would be 13.7 new babies (which is unfortunate for that .7 baby!); in a group of 2000 people, there would be 26.14 new babies; and in a group of 4 billion, a LOT of babies.

Birth rate is just one side of the equation. According to the same CIA data, the *population growth* rate in China is 0.629%. That's because it includes both newborn data and deaths.

In our population model, a 0.629% population growth means that our equation of HAVE = HAD + CHANGE, we calculate CHANGE by the expression:

CHANGE = HAD x 0.00629

Converting this into a Vensim model then becomes very easy.



The above image shows the Vensim model for "HAVE=HAD+CHANGED" in the context of population growth. Change in the population is calculated by "Growth Rate" x "Population".

Putting in some typical numbers yields a graph that resembles the following:

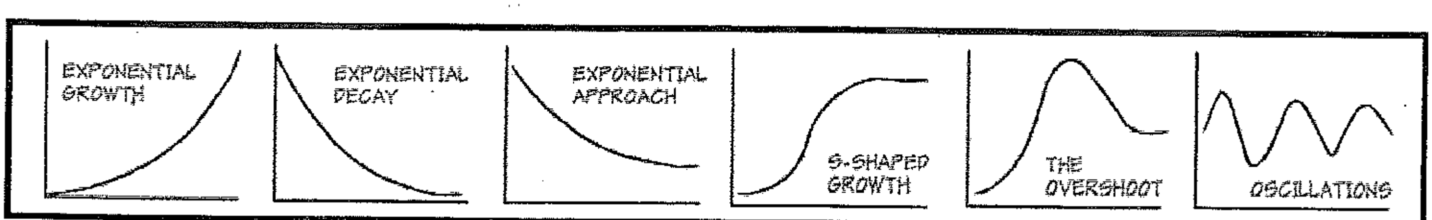


Fig. 1.3. Six shapes to represent dynamic patterns simulated in the book.

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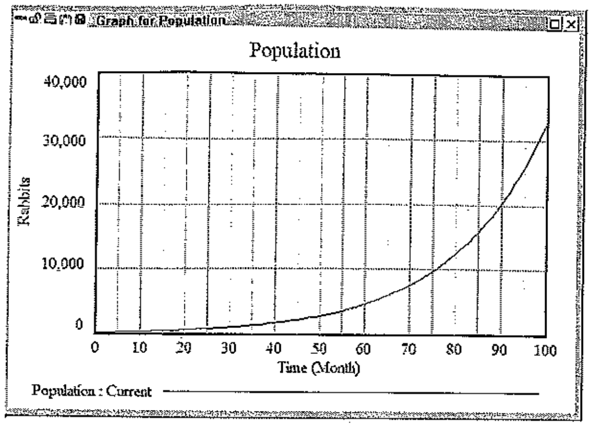

This graph illustrates the common phenomena of population growth without limits. In the absence of any factors that would limit growth, the growth of the population is exponential.

To expand on this model, consider an environment that would limit the growth of a population. Consider a rabbit population on an acreage. One thing that would limit the number of rabbits that could exist on this acreage is the availability of a food source, such as grass.

We would need to adjust our model in such a way that when the population is small, the growth rate of the population is similar to the original model. But when the population is large, the growth rate should be small, or even change from positive growth to negative growth is there are too many for the environment to sustain.

One way of representing this is with the following:
r = growth rate in the original model
P = Population size (in rabbits)
C = maximum that the environment cat support (our carrying capacity, in rabbits)

When the population is small compared to the carrying capacity, the fraction P/C is also small. When the population is close to the carrying capacity, the ratio P/C is "close to" 1. When the population is more than the carrying capacity, the ratio P/C is larger than 1.

Using these three observations, we can generate a carrying capacity growth rate with the equation:

$$r(1-P/C)$$

Using this equation, we have the properties that when P is small compared to C, the product is close to r; when P is close to C, the product is close to 0; and when P is much larger than C, the product is negative.

If we start with a population lower than the carrying capacity of the environment, using Vensim to simulate our model, the graph looks somewhat like this:

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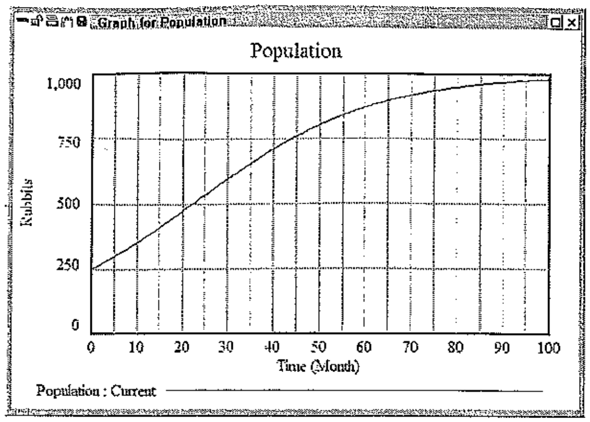
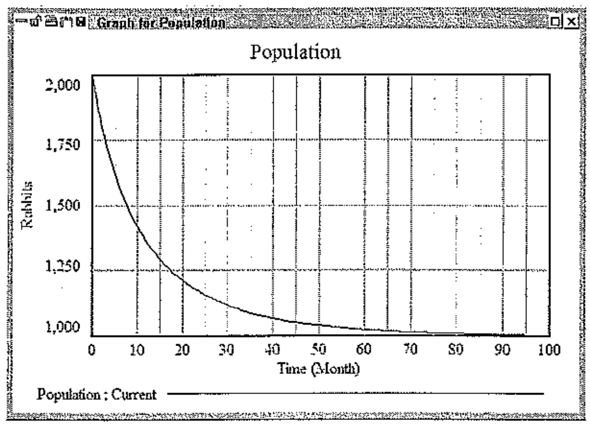
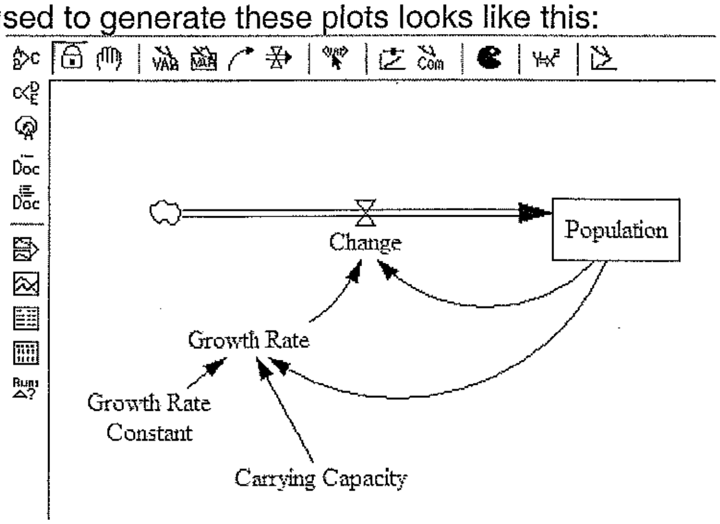


If we start with a population higher than the carrying capacity of the environment, using Vensim to simulate our model, the graph looks somewhat like this:



Note that the scale on the y-axis is different on the above plot. The lowest portion of the graph on the y-axis is the value Population=1000, which is precisely the carrying capacity of our environment in this model.

The Vensim model used to generate these plots looks like this:



Note that the Population feeds into the Growth Rate calculation, and we've added a Carrying Capacity term, and a Growth Rate Constant. The "Growth Rate" is computed as

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Growth Rate Constant * (1 - Population/Carrying Capacity)

The units are 1/Month. The units on the "Growth Rate Constant" are also 1/Month, and the unit on the "Carrying Capacity" is Rabbits.