

11.10. *You too.* Much of what I know, or imagine to know, about problem solving came to me through reflecting on relatively few suggestive experiences. In reading a book, having a discussion with a friend, talking with a student, or

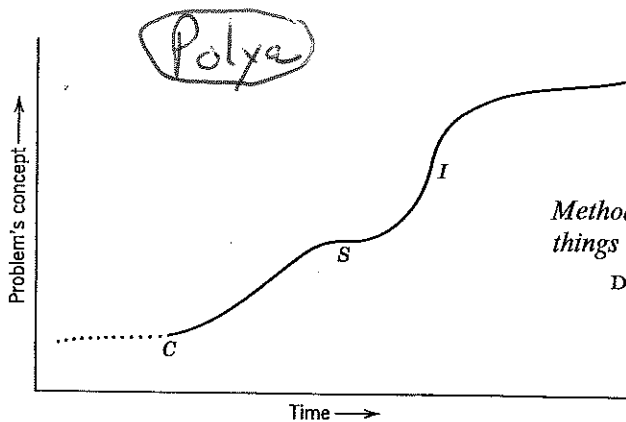


Fig. 11.4. Conscious beginning—stagnation—idea, inspiration, point of inflection.

observing the faces in an audience I suddenly recognized something and was tempted to say to myself: "You too, you are acting very much the same way." In fact I sometimes had this feeling "you too" in observing animals: dogs, birds, and once a mouse.

MICE

11.11. *Mice and men.* The landlady hurried into the backyard, put the mouse-trap on the ground (it was an old-fashioned trap, a cage with a trapdoor) and called to her daughter to fetch the cat. The mouse in the trap seemed to understand the gist of these proceedings; he raced frantically in his cage, threw himself violently against the bars, now on this side and then on the other, and in the last moment he succeeded in squeezing himself through and disappeared in the neighbour's field. There must have been on that side one slightly wider opening between the bars of the mousetrap. The landlady looked disappointed, and so did the cat who arrived too late. My sympathy from the beginning was with the

mouse, and so I found it difficult to say something polite to the landlady, or to the cat; but I silently congratulated the mouse. He solved a great problem, and gave a great example.

That is the way to solve problems. We must try and try again until eventually we recognize the slight difference between the various openings on which everything depends. We must vary our trials so that we may explore all sides of the problem. Indeed, we cannot know in advance on which side is the only practicable opening where we can squeeze through.

The fundamental method of mice and men is the same; to try, try again, and to vary the trials so that we do not miss the few favorable possibilities. It is true that men are usually better in solving problems than mice. A man need not throw himself bodily against the obstacle, he can do so mentally; a man can vary his trials more and learn more from the failure of his trials than a mouse.

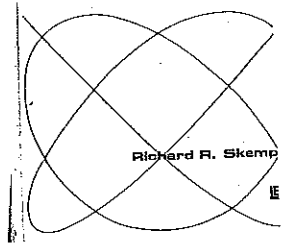
Polya

11.1. How we think

A problem solver must know his mind and an athlete must know his body in about the same way as a jockey knows his horses. I imagine that a jockey studies horses not for the sake of pure science but to make them perform better, and that he studies more the habits and whims of individual horses than horse physiology or horse psychology in general.

What you start reading now is not a chapter in a textbook of psychology; it is not exactly a conversation between problem solvers who talk about the habits of their minds as jockeys may talk about the habits of their horses; it is, however, more like a conversation than a formal presentation.

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Method consists entirely in properly ordering and arranging the things to which we should pay attention.

DESCARTES: *Cœuvres*, vol. X, p. 379; Rules for the Direction of the Mind, Rule V.

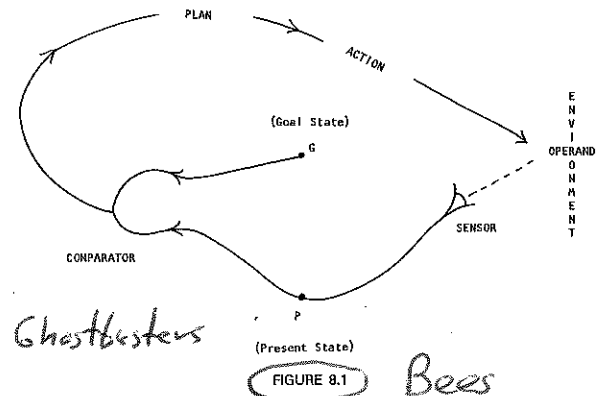
DIRECTOR SYSTEMS

Skemp

The starting point of this model was the assumption that many, perhaps most, of our actions are systematically directed toward bringing about goal states. Now, the situations in which we find ourselves doing this are not always the same. Nevertheless, we are able to achieve our goals by varying our actions appropriately, and this ability is in itself very pro-survival. A system for doing this I call a *director system*, and because the idea is borrowed from cybernetics, the ways in which a director system works will be familiar to many readers. But I need to recapitulate them briefly for the sake of what comes after.

Its essence is a comparison between the present state of some operand in the environment and its goal state, combined with a plan of action directed always so as to reduce this difference until the present state coincides with the goal state. This means that both the present state and the goal state have to be represented within the director system in some way; otherwise, how can they be compared? So, in slightly greater detail:

We need a *sensor*, which takes in information about the present state of the operand, and represents it internally. In Fig. 8.1 it is represented by an eye.
We need an internal representation of the *goal state*.



We need a *comparator*, to compare these.

And we need a *plan of action*: what we actually do to change the state of the operand from its present state to the goal state.

The distinction between the possible sources of these plans of action, and their different resulting natures, is a key feature of the present model of intelligence.

TEACHERS AND LEARNERS

A human child is at the most learning age of the most learning species that has yet evolved on this planet.

We learn in a variety of ways. Some of these, such as operant conditioning and habit learning, we share with other animals. But one of these ways is unique to man—to *homo sapiens*: not just the naked ape, as a zoologist (Morris, 1967) has called us, but the understanding ape—the ape with the potential for knowledge, even wisdom. The extent to which this potential is realised will depend, for each individual, on how he develops the intelligence with which he is born, just by virtue of being human.

This, in turn, will largely depend on his teachers. These begin with his informal teachers, such as his parents, grandparents, older brothers, and sisters. Soon, and importantly, these come to include professional teachers.

Some of the best teaching is indirect (e.g., by providing interesting activities for children to explore together). So by teaching, I mean any kind of action that is intended to influence the learning process. This process is inaccessible to direct observation by an outside person in the same way as our internal bodily processes are to a medical practitioner. In both these cases, a person who intervenes without an adequate mental image of what is going on inside is as likely to do harm as good.