CHAPTER 19 SUMMARY

The data in a **two-sample problem** are two independent SRSs, each drawn from a separate population.

Tests and confidence intervals for the difference between the means μ_1 and μ_2 of two Normal populations start from the difference $\bar{x}_1 - \bar{x}_2$ between the two sample means. Because of the central limit theorem, the resulting procedures are approximately correct for other population distributions when the sample sizes are large.

Draw independent SRSs of sizes n_1 and n_2 from two Normal populations with parameters μ_1, σ_1 and μ_2, σ_2. The **two-sample t statistic** is

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Note: usually H_0 is that $\mu_1 = \mu_2$, so $\mu_1 - \mu_2 = 0$.

The statistic t has approximately a t distribution.

Thus

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

is used.

df

There are two choices for the **degrees of freedom** of the two-sample t statistic. Option 1: software produces accurate probability values using degrees of freedom calculated from the data. Option 2: for conservative inference procedures, use degrees of freedom equal to the smaller of $n_1 - 1$ and $n_2 - 1$.

$df = \min(n_1, n_2) - 1$

We use Option 2 unless letting SPSS or SAS or Excel or R software tell us.

The **confidence interval for** $\mu_1 - \mu_2$ is

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

The critical value t^* from Option 1 gives a confidence level very close to the desired level C. Option 2 produces a margin of error at least as wide as is needed for the desired level C.

Significance tests for H_0: $\mu_1 = \mu_2$ are based on

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

P-values calculated from Option 1 are very accurate. Option 2 P-values are always at least as large as the true P.

Option 2 used on test: $df = \min(n_1 - 1, n_2 - 1)$ and look up p-value in TABLE C, t-distribution CRITICAL VALUES.