SECTION 6.1 REVIEW

TECHNIQUES

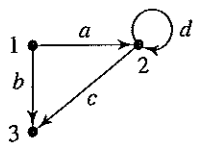
- Use graph terminology.
- Prove or disprove that two graphs are isomorphic.
- Find a planar representation of a simple graph or prove that none exists.
- Construct adjacency matrices and adjacency lists for graphs and directed graphs.

MAIN IDEAS

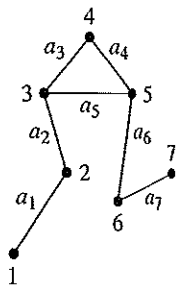
- Diverse situations can be modeled by graphs.
- Graphs can be represented in a computer by matrices or by linked lists.

EXERCISES 6.1

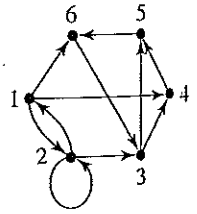
1. Give the function g that is part of the formal definition of the directed graph shown.



2. Use the graph in the figure to answer the questions that follow.
   a. Is the graph simple?
   b. Is the graph complete?
   c. Is the graph connected?
   d. Can you find two paths from 3 to 6?
   e. Can you find a cycle?
   f. Can you find an arc whose removal will make the graph acyclic?
   g. Can you find an arc whose removal will make the graph not connected?

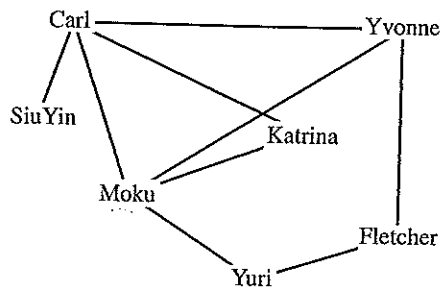


3. Sketch a picture of each of the following graphs.
   a. Simple graph with three nodes, each of degree 2
   b. Graph with four nodes, with cycles of length 1, 2, 3, and 4
   c. Noncomplete graph with four nodes, each of degree 4
4. Use the directed graph in the figure to answer the questions that follow.
   a. Which nodes are reachable from node 3?
   b. What is the length of the shortest path from node 3 to node 6?
   c. What is a path from node 1 to node 6 of length 8?



5. Draw K_6.
6. Draw $K_{3,4}$.
7. For each of the following characteristics, draw a graph or explain why such a graph does not exist.
   a. Four nodes of degree 1, 2, 3, and 4, respectively
   b. Simple, four nodes of degree 1, 2, 3, and 4, respectively
   c. Four nodes of degree 2, 3, 3, and 4, respectively
   d. Four nodes of degree 2, 3, 3, and 3, respectively

8. For each of the following characteristics, draw a graph or explain why such a graph does not exist.
   a. Simple graph with seven nodes, each of degree 3
   b. Four nodes, two of degree 2 and two of degree 3
   c. Three nodes of degree 0, 1, and 3, respectively
   d. Complete graph with 4 nodes each of degree 2
9. An *acquaintanceship graph* is an undirected graph in which the nodes represent people and nodes a and b are adjacent if a and b are acquainted.
   a. The acquaintanceship graph for the IT department and the marketing department of a major corporation is an unconnected graph. What does this imply?
   b. The following figure represents an acquaintanceship graph for residents of an apartment building. Are Carl and Fletcher acquainted? How many people is SiuYin acquainted with?



   c. The length of the shortest path between node a and node b in an acquaintanceship graph is sometimes called the *degree of separation* between a and b. What is the degree of separation between Carl and Yuri?
10. The "small world effect" states that the average degree of separation (see Exercise 9) in an acquaintanceship graph of the whole world is 6. In other words, a path of acquaintance relationships from you to any other person on earth exists with, on the average, a path length of 6 (5 intermediate persons). Experiments in delivering hard-copy letters and e-mail messages have empirically confirmed this theory.[1]
    a. What are the potential implications for e-mail traffic if the small world effect holds for computer networks?
    b. What are the potential implications for epidemiology if the small world effect holds for physical contact between humans?
11. The small world effect (see Exercise 10) has been found to be true between root words (that is, basic words found in a thesaurus) in the English language, with an average degree of separation equal to 3. Here "adjacent words" are those that are listed as synonyms in an English thesaurus. For example, "gate" and "commotion" are related by 3 degrees of separation, as follows:

$$\text{gate} \rightarrow \text{door} \rightarrow \text{flap} \rightarrow \text{commotion}$$

    Can you think of 3 degrees of separation between the following pairs of words?
    a. "star" and "sculpture"
    b. "burden" and "influence"
    c. "piano" and "significance"

[1]But more recent analyses of 721 million Facebook users, a much larger community than was available to earlier studies, suggests that the average number of intermediaries between persons A and B is 3.74. It's a small world indeed, at least for Facebook users.

12. An idea closely related to the average degree of separation in a graph is that of clustering. The *global clustering coefficient* for a given graph is given by

$$C = \frac{3 * T}{t}$$

where T = the number of triangles in the graph and t = the number of connected node triples.

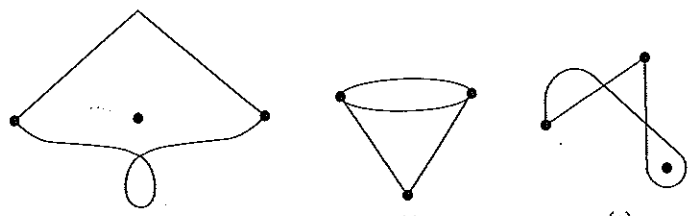
A connected node triple is a "center" node adjacent to an unordered pair of other nodes. For example, in the graph of Exercise 2, 3–4–5 (or 5–4–3) and 4–5–6 (6–5–4) are two such triples. Nodes that make up a triangle demonstrate transitivity; if a is adjacent to b and b is adjacent to c, then a is adjacent to c. Therefore c is a ratio of nodes in a transitive threesome to all nodes in a threesome. (One might think of this in terms of a social network as the probability that if you are a "friend" of mine and x is a "friend" of yours, then x is also a "friend" of mine.)

a. Consider the graph in Figure 6.28 and the graph for Exercise 2. Which do you think has the higher clustering coefficient?

b. Compute the clustering coefficient for the graph in Figure 6.28

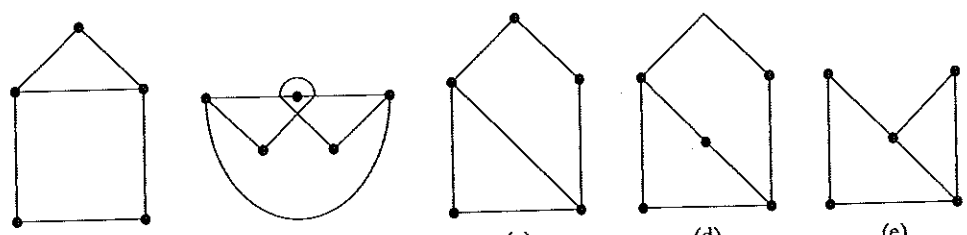
c. Compute the clustering coefficient for the graph for Exercise 2.

13. Which of the following graphs is not isomorphic to the others, and why?



(a) (b) (c)

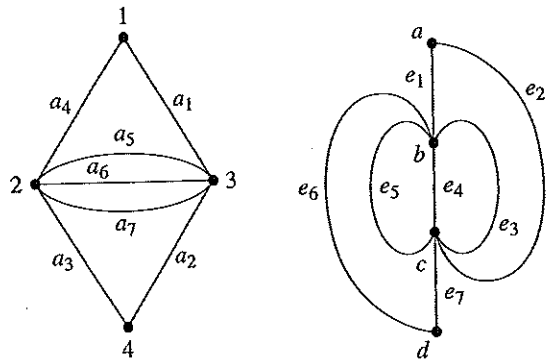
14. Which of the following graphs is not isomorphic to the others, and why?



(a) (b) (c) (d) (e)

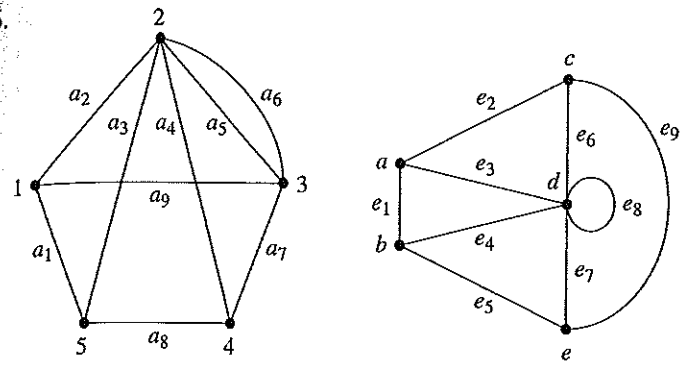
For Exercises 15–20, decide if the two graphs are isomorphic. If so, give the function or functions that establish the isomorphism; if not, explain why.

15.



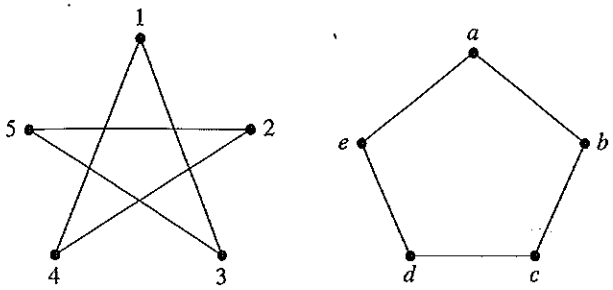
(a) (b)

16.



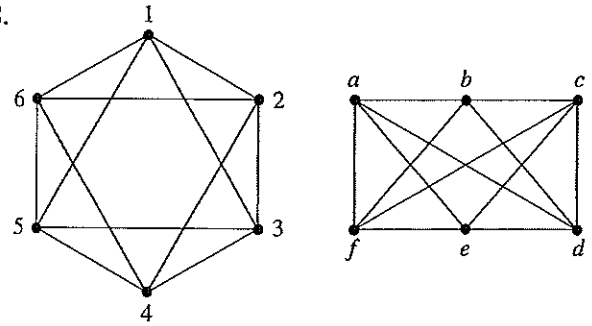
(a) (b)

17.



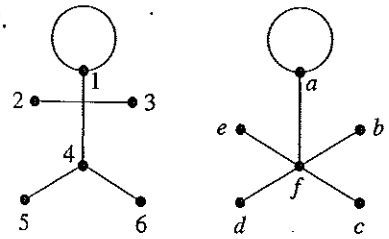
(a) (b)

18.



(a) (b)

19.



(a) (b)