Exercise Set 1.2

1. Which of the following sets are equal?
   \[ A = \{a, b, c, d\} \quad B = \{d, e, a, c\} \]
   \[ C = \{d, b, a, c\} \quad D = \{a, a, d, e, c, e\} \]

2. Write in words how to read each of the following out loud.
   a. \( \{x \in \mathbb{R}^2 \mid 0 < x < 1\} \)
   b. \( \{x \in \mathbb{R} \mid x \leq 0 \text{ or } x \geq 1\} \)
   c. \( \{n \in \mathbb{Z} \mid n \text{ is a factor of } 6\} \)
   d. \( \{n \in \mathbb{Z}^+ \mid n \text{ is a factor of } 6\} \)

3. a. Is \( 4 = \{4\} \)?
   b. How many elements are in the set \( \{3, 4, 3, 5\} \)?
   c. How many elements are in the set \( \{1, [1], [1], [1] \} \)?

4. a. Is \( 2 \in \{2\} \)?
   b. How many elements are in the set \( \{2, 2, 2, 2\} \)?
   c. How many elements are in the set \( \{0, [0] \} \)?
   d. Is \( \{0\} \in \{[0], \{1\}\} \)?
   e. Is \( 0 \in \{[0], \{1\}\} \)?

H 5. Which of the following sets are equal?
   \[ A = \{0, 1, 2\} \]
   \[ B = \{x \in \mathbb{R} \mid -1 \leq x < 3\} \]
   \[ C = \{x \in \mathbb{R} \mid -1 < x < 3\} \]
   \[ D = \{x \in \mathbb{Z} \mid -1 \leq x < 3\} \]
   \[ E = \{x \in \mathbb{Z}^+ \mid -1 < x < 3\} \]

H 6. For each integer \( n \), let \( T_n = [n, n^2] \). How many elements are in each of \( T_2, T_{-3}, T_1 \) and \( T_0 \)? Justify your answers.

7. Use the set-roster notation to indicate the elements in each of the following sets.
   a. \( S = \{n \in \mathbb{Z} \mid n = (-1)^k, \text{ for some integer } k\} \)
   b. \( T = \{m \in \mathbb{Z} \mid m = 1 + (-1)^i, \text{ for some integer } i\} \)

8. Let \( A = \{c, d, f, g\} \), \( B = \{f, j\} \), and \( C = \{d, g\} \). Answer each of the following questions. Give reasons for your answers.
   a. Is \( B \subseteq A \)?
   b. Is \( C \subseteq A \)?
   c. Is \( C \subseteq B \)?
   d. Is \( C \) a proper subset of \( A \)?

9. a. Is \( 3 \in \{1, 2, 3\} \)?
   b. Is \( 1 \in \{1\} \)?
   c. Is \( 2 \in \{1, 2\} \)?
   d. Is \( 3 \in \{1, 2, 3\} \)?
   e. Is \( 1 \in \{1\} \)?
   f. Is \( 2 \in \{1, 2, 3\} \)?
   g. Is \( 1 \in \{1, 2\} \)?
   h. Is \( 1 \in \{\{1\}, 2\} \)?
   i. Is \( 1 \in \{\{1\}, 2\} \)?
   j. Is \( 1 \in \{1\} \)?

10. a. Is \( (\sqrt{-2})^2, -2^2 = (-2^2, -2^2) \)?
    b. Is \( (5, -5) \equiv (-5, 5) \)?
    c. Is \( (8 - 9, \sqrt{-1}) = (-1, -1) \)?
    d. Is \( \left( \frac{\sqrt{2}}{4}, (-2)^1 \right) = \left( \frac{3}{6}, -8 \right) \)?

11. Let \( A = \{w, x, y, z\} \) and \( B = \{a, b\} \). Use the set-roster notation to write each of the following sets, and indicate the number of elements that are in each set:
   a. \( A \times B \)
   b. \( B \times A \)
   c. \( A \times A \)
   d. \( B \times B \)

12. Let \( S = \{2, 4, 6\} \) and \( T = \{1, 3, 5\} \). Use the set-roster notation to write each of the following sets, and indicate the number of elements that are in each set:
   a. \( S \times T \)
   b. \( T \times S \)
   c. \( S \times S \)
   d. \( T \times T \)
3. Let sets $R$, $S$, and $T$ be defined as follows:

$R = \{x \in \mathbb{Z} \mid x \text{ is divisible by } 2\}$

$S = \{y \in \mathbb{Z} \mid y \text{ is divisible by } 3\}$

$T = \{z \in \mathbb{Z} \mid z \text{ is divisible by } 6\}$

a. Is $R \subseteq T$? Explain.

b. Is $T \subseteq R$? Explain.

c. Is $T \subseteq S$? Explain.

4. Let $A = \{x \in \mathbb{Z} \mid n = 5r \text{ for some integer } r\}$ and $B = \{m \in \mathbb{Z} \mid m = 20s \text{ for some integer } s\}$.

a. Is $A \subseteq B$? Explain.

b. Is $B \subseteq A$? Explain.

5. Let $C = \{x \in \mathbb{Z} \mid n = 6r - 4 \text{ for some integer } r\}$ and $D = \{m \in \mathbb{Z} \mid m = 3s + 1 \text{ for some integer } s\}$.

Prove or disprove each of the following statements.

a. $C \subseteq D$

b. $D \subseteq C$

6. Let $A = \{x \in \mathbb{Z} \mid x = 5a + 2 \text{ for some integer } a\}$, $B = \{y \in \mathbb{Z} \mid y = 10b - 3 \text{ for some integer } b\}$, and $C = \{z \in \mathbb{Z} \mid z = 10c + 7 \text{ for some integer } c\}$.

Prove or disprove each of the following statements.

a. $A \subseteq B$

b. $B \subseteq A$

c. $B = C$

7. Let $A = \{x \in \mathbb{Z} \mid x = 6a + 4 \text{ for some integer } a\}$, $B = \{y \in \mathbb{Z} \mid y = 18b - 2 \text{ for some integer } b\}$, and $C = \{z \in \mathbb{Z} \mid z = 18c + 16 \text{ for some integer } c\}$.

Prove or disprove each of the following statements.

a. $A \subseteq B$

b. $B \subseteq A$

c. $B = C$

8. Write in words how to read each of the following out loud.

Then write the shorthand notation for each set.

a. $\{x \in U \mid x \in A \text{ and } x \notin B\}$

b. $\{x \in U \mid x \in A \text{ or } x \notin B\}$

c. $\{x \in U \mid x \in A \} \cap \{x \notin B\}$

d. $\{x \in U \mid x \notin A\}$

9. Complete the following sentences without using the symbols $\cup$, $\cap$, or $\setminus$.

a. $x \notin A \cup B$, if and only if, _____.

b. $x \notin A \cap B$, if and only if, _____.

c. $x \notin A - B$, if and only if, _____.

10. Let $A = \{1, 3, 5, 7\}$, $B = \{3, 6, 9\}$, and $C = \{2, 4, 6, 8\}$.

Find each of the following:

a. $A \cup B$

b. $A \cap B$

c. $A \cup C$

d. $A \cap C$

e. $A - B$

f. $B - A$

g. $B \cup C$

h. $B \cap C$

11. Let the universal set be the set $\mathbb{R}$ of all real numbers and let $A = \{x \in \mathbb{R} \mid 0 < x \leq 2\}$, $B = \{x \in \mathbb{R} \mid 1 \leq x < 4\}$, and $C = \{x \in \mathbb{R} \mid 3 \leq x < 9\}$.

Find each of the following:

a. $A \cup B$

b. $A \cap B$

c. $A' \cup B$

d. $A \cup C$

e. $A \cap C$

f. $B' \cup C$

g. $A' \cap B'$

h. $(A \cup B)'$

i. $(A \cup C)'$

j. $(A \cap B)'$

12. Let the universal set be the set $\mathbb{R}$ of all real numbers and let $A = \{x \in \mathbb{R} \mid -3 \leq x \leq 0\}$, $B = \{x \in \mathbb{R} \mid -1 < x < 2\}$, and $C = \{x \in \mathbb{R} \mid 5 < x \leq 8\}$.

Find each of the following:

a. $A \cup B$

b. $A \cap B$

c. $A' \cup B$

d. $A \cup C$

e. $A \cap C$

f. $B' \cup C$

g. $(A' \cap B')$

h. $(A \cup B)'$

i. $(A \cap B)'$

13. Indicate which of the following relationships are true and which are false:

a. $Z^+ \subseteq Q$

d. $R^- \subseteq Q$

b. $R^- \subseteq Q$

e. $Q \subseteq Z$

g. $Q \subseteq Z$

b. $Q \subseteq Z$

c. $Z^+ \cap Z^+ = Z$

f. $Q \cap R = Q$

g. $Q \cap R = Q$

h. $Z^+ \cap R = Z^+$

t. $Z \cap Q = Z$

14. In each of the following, draw a Venn diagram for sets $A$, $B$, and $C$ that satisfy the given conditions:

a. $A \subseteq B; C \subseteq B; A \cap C = \emptyset$

b. $C \subseteq A; B \cap C = \emptyset$

15. Draw Venn diagrams to describe sets $A$, $B$, and $C$ that satisfy the given conditions.

a. $A \cap B = \emptyset, A \subseteq C, C \cap B \neq \emptyset$

b. $A \subseteq C, B \subseteq C, A \cap C \neq \emptyset$

c. $A \cap C \neq \emptyset, B \cap C \neq \emptyset, A \cap C = \emptyset, A \nsubseteq B, C \nsubseteq B$

16. Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, and $C = \{b, c, e\}$.

a. Find $A \cup (B \cap C)$, $(A \cup B) \cap C$, and $(A \cup B) \cap (A \cup C)$. Which of these sets are equal?

b. Find $A \cap (B \cup C)$, $(A \cap B) \cup C$, and $(A \cap B) \cup (A \cap C)$. Which of these sets are equal?

c. Find $(A - B) - C$ and $A - (B - C)$. Are these sets equal?

17. Consider the Venn diagram shown below. For each of (a)-(f), copy the diagram and shade the region corresponding to the indicated set.

a. $A \cap B$

b. $B \cup C$

c. $A'$

d. $A - (B \cup C)$

e. $(A \cup B)'$

f. $A' \cap B'$

18. a. Is the number 0 in $\emptyset$? Why?

b. Is $\emptyset = \{0\}$? Why?

c. Is $\emptyset \in \{0\}$? Why?

d. Is $0 \in \emptyset$? Why?

19. Let $A_i = \{i, i^2\}$ for all integers $i = 1, 2, 3, 4$.

a. $A_1 \cup A_2 \cup A_3 \cup A_4 = ?$

b. $A_1 \cap A_2 \cap A_3 \cap A_4 = ?$

c. Are $A_1, A_2, A_3,$ and $A_4$ mutually disjoint? Explain.

20. Let $B_i = \{x \in \mathbb{R} \mid 0 \leq x \leq i\}$ for all integers $i = 1, 2, 3, 4$.

a. $B_1 \cup B_2 \cup B_3 \cup B_4 = ?$

b. $B_1 \cap B_2 \cap B_3 \cap B_4 = ?$

c. Are $B_1, B_2, B_3,$ and $B_4$ mutually disjoint? Explain.

21. Let $C_i = [i, -i]$ for all nonnegative integers $i$.

a. $\bigcup_{i=0}^{\infty} C_i = ?$

b. $\bigcap_{i=0}^{\infty} C_i = ?$