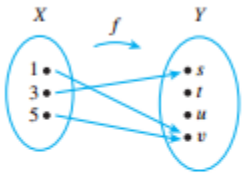
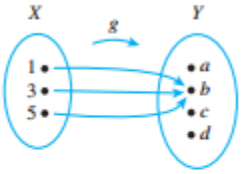
This textbook uses the term "co-domain" where your textbook used the term "target"

1. Let $X = \{1, 3, 5\}$ and $Y = \{s, t, u, v\}$. Define $f: X \to Y$ by the following arrow diagram.



a. Write the domain of f and the co-domain of f.
b. Find $f(1)$, $f(3)$, and $f(5)$.
c. What is the range of f?
d. Is 3 an inverse image of s? Is 1 an inverse image of u?
e. What is the inverse image of s? of u? of v?
f. Represent f as a set of ordered pairs.

2. Let $X = \{1, 3, 5\}$ and $Y = \{a, b, c, d\}$. Define $g: X \to Y$ by the following arrow diagram.



a. Write the domain of g and the co-domain of g.
b. Find $g(1)$, $g(3)$, and $g(5)$.
c. What is the range of g?
d. Is 3 an inverse image of a? Is 1 an inverse image of b?
e. What is the inverse image of b? of c?
f. Represent g as a set of ordered pairs.

3. Indicate whether the statements in parts (a)–(d) are true or false. Justify your answers.
a. If two elements in the domain of a function are equal, then their images in the co-domain are equal.
b. If two elements in the co-domain of a function are equal, then their preimages in the domain are also equal.
c. A function can have the same output for more than one input.
d. A function can have the same input for more than one output.

4. a. Find all functions from $X = \{a, b\}$ to $Y = \{u, v\}$.
b. Find all functions from $X = \{a, b, c\}$ to $Y = \{u\}$.
c. Find all functions from $X = \{a, b, c\}$ to $Y = \{u, v\}$.

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1. The definition of one-to-one is stated in two ways:

$$\forall x_1, x_2 \in X, \text{ if } F(x_1) = F(x_2) \text{ then } x_1 = x_2$$

and $$\forall x_1, x_2 \in X, \text{ if } x_1 \neq x_2 \text{ then } F(x_1) \neq F(x_2).$$

Why are these two statements logically equivalent?

2. Fill in each blank with the word *most* or *least*.
a. A function F is one-to-one if, and only if, each element in the co-domain of F is the image of at _____ one element in the domain of F.
b. A function F is onto if, and only if, each element in the co-domain of F is the image of at _____ one element in the domain of F.

f 3. When asked to state the definition of one-to-one, a student replies, "A function f is one-to-one if, and only if, every element of X is sent by f to exactly one element of Y." Give a counterexample to show that the student's reply is incorrect.

H 4. Let $f: X \to Y$ be a function. True or false? A sufficient condition for f to be one-to-one is that for all elements y in Y, there is at most one x in X with $f(x) = y$.

H 5. All but two of the following statements are correct ways to express the fact that a function f is onto. Find the two that are incorrect.
a. f is onto $\Leftrightarrow$ every element in its co-domain is the image of some element in its domain.
b. f is onto $\Leftrightarrow$ every element in its domain has a corresponding image in its co-domain.
c. f is onto $\Leftrightarrow \forall y \in Y, \exists x \in X$ such that $f(x) = y$.
d. f is onto $\Leftrightarrow \forall x \in X, \exists y \in Y$ such that $f(x) = y$.
e. f is onto $\Leftrightarrow$ the range of f is the same as the co-domain of f.

6. Let $X = \{1, 5, 9\}$ and $Y = \{3, 4, 7\}$.
a. Define $f: X \to Y$ by specifying that

$$f(1) = 4, \quad f(5) = 7, \quad f(9) = 4.$$

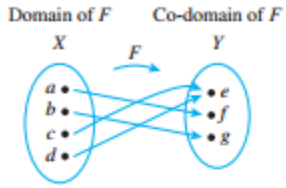
Is f one-to-one? Is f onto? Explain your answers.

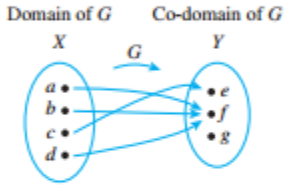
b. Define $g: X \to Y$ by specifying that

$$g(1) = 7, \quad g(5) = 3, \quad g(9) = 4.$$

Is g one-to-one? Is g onto? Explain your answers.

7. Let $X = \{a, b, c, d\}$ and $Y = \{e, f, g\}$. Define functions F and G by the arrow diagrams below.

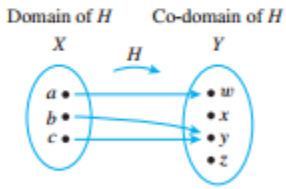


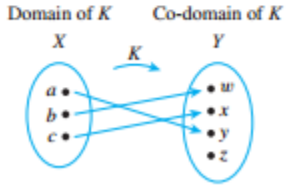


a. Is F one-to-one? Why or why not? Is it onto? Why or why not?

b. Is G one-to-one? Why or why not? Is it onto? Why or why not?

8. Let $X = \{a, b, c\}$ and $Y = \{w, x, y, z\}$. Define functions H and K by the arrow diagrams below.





a. Is H one-to-one? Why or why not? Is it onto? Why or why not?

b. Is K one-to-one? Why or why not? Is it onto? Why or why not?

10. a. Define $f: \mathbf{Z} \to \mathbf{Z}$ by the rule $f(n) = 2n$, for all integers n.

(i) Is f one-to-one? Prove or give a counterexample.

(ii) Is f onto? Prove or give a counterexample.

b. Let $2\mathbf{Z}$ denote the set of all even integers. That is, $2\mathbf{Z} = \{n \in \mathbf{Z} \mid n = 2k, \text{ for some integer } k\}$. Define $h: \mathbf{Z} \to 2\mathbf{Z}$ by the rule $h(n) = 2n$, for all integers n. Is h onto? Prove or give a counterexample.

11. a. Define $g: \mathbf{Z} \to \mathbf{Z}$ by the rule $g(n) = 4n - 5$, for all integers n.

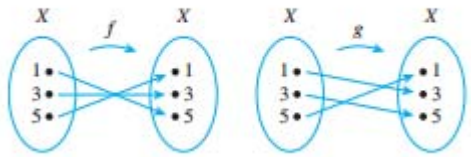
(i) Is g one-to-one? Prove or give a counterexample.

(ii) Is g onto? Prove or give a counterexample.

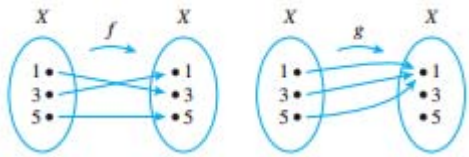
b. Define $G: \mathbf{R} \to \mathbf{R}$ by the rule $G(x) = 4x - 5$ for all real numbers x. Is G onto? Prove or give a counterexample.

For problems 1-4, find G ®F and F®G and determine whether G ®F = F®G

1.



2.



3. $F(x) = x^3$ and $G(x) = x - 1$, for all real numbers x.

4. $F(x) = x^5$ and $G(x) = x^{1/5}$ for all real numbers x.

5. Define $f: \mathbf{R} \to \mathbf{R}$ by the rule $f(x) = -x$ for all real numbers x. Find $(f \circ f)(x)$.

6. Define $F: \mathbf{Z} \to \mathbf{Z}$ and $G: \mathbf{Z} \to \mathbf{Z}$ by the rules $F(a) = 7a$ and $G(a) = a \bmod 5$ for all integers a. Find $(G \circ F)(0)$, $(G \circ F)(1)$, $(G \circ F)(2)$, $(G \circ F)(3)$, and $(G \circ F)(4)$.

7. Define $H: \mathbf{Z} \to \mathbf{Z}$ and $K: \mathbf{Z} \to \mathbf{Z}$ by the rules $H(a) = 6a$ and $K(a) = a \bmod 4$ for all integers a. Find $(K \circ H)(0)$, $(K \circ H)(1)$, $(K \circ H)(2)$, and $(K \circ H)(3)$.

8. Define $L: \mathbf{Z} \to \mathbf{Z}$ and $M: \mathbf{Z} \to \mathbf{Z}$ by the rules $L(a) = a^2$ and $M(a) = a \bmod 5$ for all integers a.
   a. Find $(L \circ M)(12)$, $(M \circ L)(12)$, $(L \circ M)(9)$, and $(M \circ L)(9)$.
   b. Is $L \circ M = M \circ L$?