Prove: For all integers if x – y is odd, the x is odd or y is odd.

PROOF:

Rather than prove the original theorem, let’s consider it’s contraposition. That is:

For all integers if x is even and y is even then x – y is even

Let’s assume that:

X and y are even integers by given.

Let x = 2k for some integer k

And y = 2j for some integer j [Definition of even numbers]

X – y = 2k - 2j [substitution]

 = 2 (k – j) [factoring/algebra/distributive property]

Let f be the integer such that, f = k – j [closure of integers on subtraction]

X – y = 2f , for some integer f [substitution[

Therefore x – y is an even number by definition of even numbers.

Since I proved the contraposition of my original statement to be true, we have also proven the original statement to be true thru proof by contraposition.

PROVE: There is no greatest integer

PROOF:

Let’s assume that the negation of this is true.

That is, let’s assume that there IS a greatest integer.

Let x be that greatest integer

We know that

X < x + 1 [rules of inequality]

Let y be the value x+1

Y = x+1 for some integer y [ closure of integers on addition]

X < y [substitution]

Therefore x is not the greatest integer

Since we proved the negation the to be false, the original theorem must be true.

* Prove: There is no integer that is both even and odd

PROOF:

Let’s assume that the negation of this is true.

That is,

There exists some integer that is both even and odd.

Let x be that integer

Since x is even

X = 2k for some integer k [def of even]

Since x is odd

X = 2j+1 for some integer j [def of odd]

2k = 2j + 1 [sub]

2k – 2j = 1

2 (k – j) = 1 [factoring]

K – j = ½

This a contradiction. Integers are closed on subtraction and can not produce a non-integer.

Since the negation is clearly false, the original statement must be true.