Prove: If 3n+2 is odd, then n is odd

PROOF

Let’s begin by considering the negation of this theorem.

That is:

3n+ 2 is odd and n is even

[Reminder, the negation of an implication is NOT implication]

We want to show that this statement is false.

Let n be an arbitrary even number. [Given]

Then n = 2k for some integer k [def of even]

3n + 2 = 3(2k) + 2 [substitution]

= 2(3k) + 2\*1 [algebra]

= 2 (3k + 1) [Distributive property

Let integer x = 3k+1 [closure of integers on \* and +]

= 2 x [substitution]

This means that 3n + 2 is even by definition.

However, this means that our new statement (our negation) is false.

Since the negation is false, the original theorem must be true. We have proven our original theorem through proof by contradiction.

* PROVE: The product of any two consecutive integers is even.

PROOF:

Let’s consider consecutive integers, X and x+1 (given)

Since all integers are either even or odd, we must consider both possibilities here. That is, that x is even or x is odd.

Case 1 : x is even

Let x and x+1 be consecutive integers where x is even

Let x = 2k for some integer k [def of even]

x \* (x+1) = 2k \* (2k + 1) [substitution]

= 2 \* (2k\*k + 1\*k) [distributive property]

Let integer y = (2k\*k + k) [closure of ints on \* and +]

= 2\*y for integer y

Therefore, the product of these two consecutive numbers is even.

Case 2 : x is odd

Let x and x+1 be consecutive integers where x is odd

Let x = 2j+1 for some integer j [def of odd]

X\*(x+1) = (2j+1)\* (2j+1 +1) [substitution]

= (2j + 1) (2j + 2) [algebra]

= 2\*2\*j\*j + 2\*j\*2 +2j\*1 + 2\*1

= 2 (2j^2 + 2j + j + 1)

Let integer z = 2j^2 + 3j +1 [closure of ints on \* and +]

x\*(x+1) = 2(z) for integer z

Therefore, the product of these two consecutive numbers is even.

For both cases of x, we have shown x\*(x+1) is even

Therefore, we have proven out theorem.

Prove x + | x – 7 | ≥ 7 for all values of x.

Let’s break this into 3 cases:

Case 1: x=7

Case 2: x>7

Case 3: x<7

Case 1:

Let x=7. [Given]

x + | x – 7 | = 7 + | 7 – 7 | [Substitution]

= 7 + | 0 |

= 7

≥ 7

Thus, we have proven the theorem for case 1.

Case 2:

Let x>7 [Given]

If x>7 then x = 7 + a for some positive value of a.

x + | x – 7 | = 7+a + | 7+a – 7 | [Substitution]

= 7 + a + | a | [Simplification]

Since we know that a is positive, we know that |a| =a [definition of absolute value]

x + | x – 7 | = 7 + a + | a |

= 7 + a + a [substitution]

= 7 + 2a

Since a is a positive value, 2a is a positive value an

7 + 2a > 7

Therefore

x + | x – 7 | = 7 + 2a > 7 ≥ 7

Thus, we have proven the theorem for case 2.

Case 3:

Let x<7 [Given]

If x<7 then x = 7 - a for some positive value of a.

x + | x – 7 | = 7-a + | 7-a – 7 | [Substitution]

= 7 - a + | a | [Simplification]

Since we know that a is positive, we know that |a| =a [definition of absolute value]

x + | x – 7 | = 7 - a + | a |

= 7 - a + a [substitution]

= 7 ≥ 7

Thus, we have proven the theorem for case 3.

Since we have proven that

x + | x – 7 | ≥ 7

for all three cases, and these thee cases cover all possible values of x, we have proven this theorem to be true.