1. **Traveling Salesperson Problem** (TSP) dynamic programming algorithm -- build-up the lengths of optimal paths ending at \( v_1 \) by growing the subset of intermediate vertices on the path, call this set \( A \). For a set of vertices \( V = \{v_1, v_2, \ldots, v_n\} \) with an adjacency matrix \( W \), define a distance matrix \( D \), such that

\[
D[v_1][A] = \text{length of the shortest path from } v_1 \text{ to } v_1 \text{ passing through every vertex in } A \text{ exactly once}
\]

a) The "smallest" size problems would be represented by an empty set (denoted by the symbol \( \emptyset \)) of intermediate vertices. For each vertex \( v_i \) in \( V - \{v_1\} \), what would we initialize the value of

\[
D[v_i][\emptyset] =
\]

b) To grow the subset \( A \) (which represent "bigger" problems) by one vertex, why do we need to consider only each vertex \( v_i \) in \( V - (A \cup \{v_1\}) \)?

c) For \( A \neq \emptyset \), complete the recursive relationship to calculate \( D[v_1][A] \) using the following diagram as a guide.

Consider paths ending in each vertex in \( A \)

Each vertex in \( A \), \( v_j \)

\[
D[v_1][A] =
\]

2. Consider the following graph with \( V = \{v_1, v_2, v_3, v_4\} \). List all the subsets of \( V - \{v_1\} \) of the following sizes:

a) 0 (empty set)

b) 1

c) 2

d) 3

3. For the above graph, what are the smallest problems in the distance matrix \( D \)?

\[
D[v_2][\emptyset] =
\]

\[
D[v_3][\emptyset] =
\]

\[
D[v_4][\emptyset] =
\]
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4. For the above graph, what are the next bigger problems with subsets of size one?
   \[ D[v_2](\{v_1\}) = \]
   \[ D[v_2](\{v_4\}) = \]
   \[ D[v_3](\{v_2\}) = \]
   \[ D[v_3](\{v_4\}) = \]
   \[ D[v_4](\{v_2\}) = \]
   \[ D[v_4](\{v_3\}) = \]

5. For the above graph, what are the next bigger problems with subsets of size two?
   \[ D[v_2](\{v_3, v_4\}) = \]
   \[ D[v_3](\{v_2, v_4\}) = \]
   \[ D[v_4](\{v_2, v_3\}) = \]

6. For the above graph, what are the next bigger problems with subsets of size three?
   \[ D[v_1](\{v_2, v_3, v_4\}) = \]

7. What is the theta notation of the following TSP algorithm?

   ```c
   void travel (int n, const number W[][], index P[][], number& minlength) {
     index i, j, k;
     number D[1..n][subset of V - \{v_i\}]
     for (i=2; i <=n; i++)
       D[i][\emptyset] = W[i][1];
     for (k = 1; k <= n-2; k++)
       for (all subsets A ⊆ V - \{v_i\} containing k vertices)
         for (i such that i ≠ 1 and v_i is not in A) {
           \[ D[i][A] = \min_{j \in A} (W[i][j] + D[j][A - \{v_j\}]) \]
         P[i][A] = value of j that gave the minimum
       }
     D[1][V - \{v_i\}] = \min_{2 \le j \le n} (W[1][j] + D[j][V - \{v_i, v_j\}]);
     P[1][V - \{v_i\}] = value of j that gave the minimum
     minlength = D[1][V - \{v_i\}];
   }
   ```